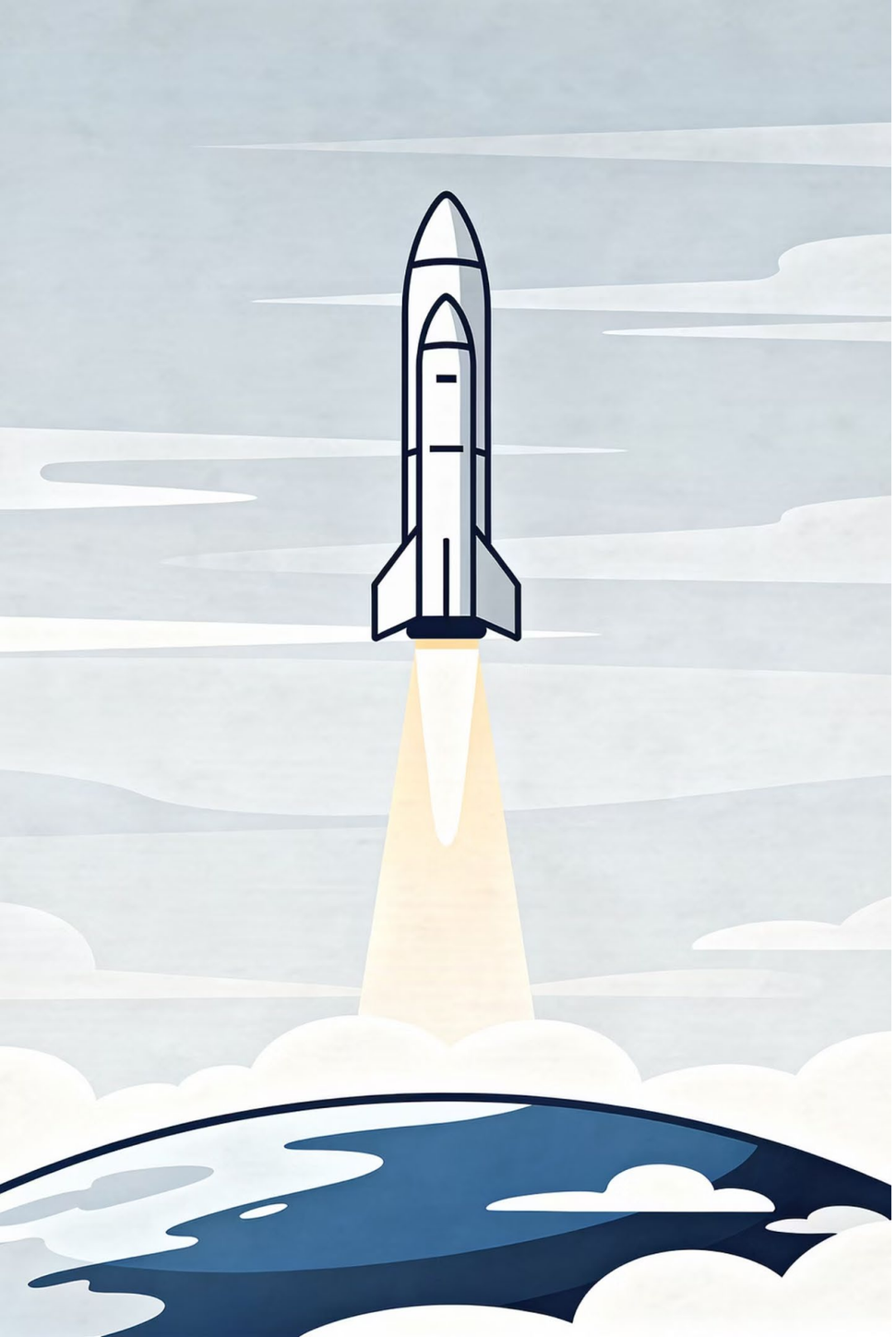


Orbits & Gravity

How do objects stay in orbit without flying away or crashing into Earth? Today we'll explore the physics that keeps satellites, the International Space Station, and even the Moon circling our planet.

Aerospace Engineering: Orbits & Gravity

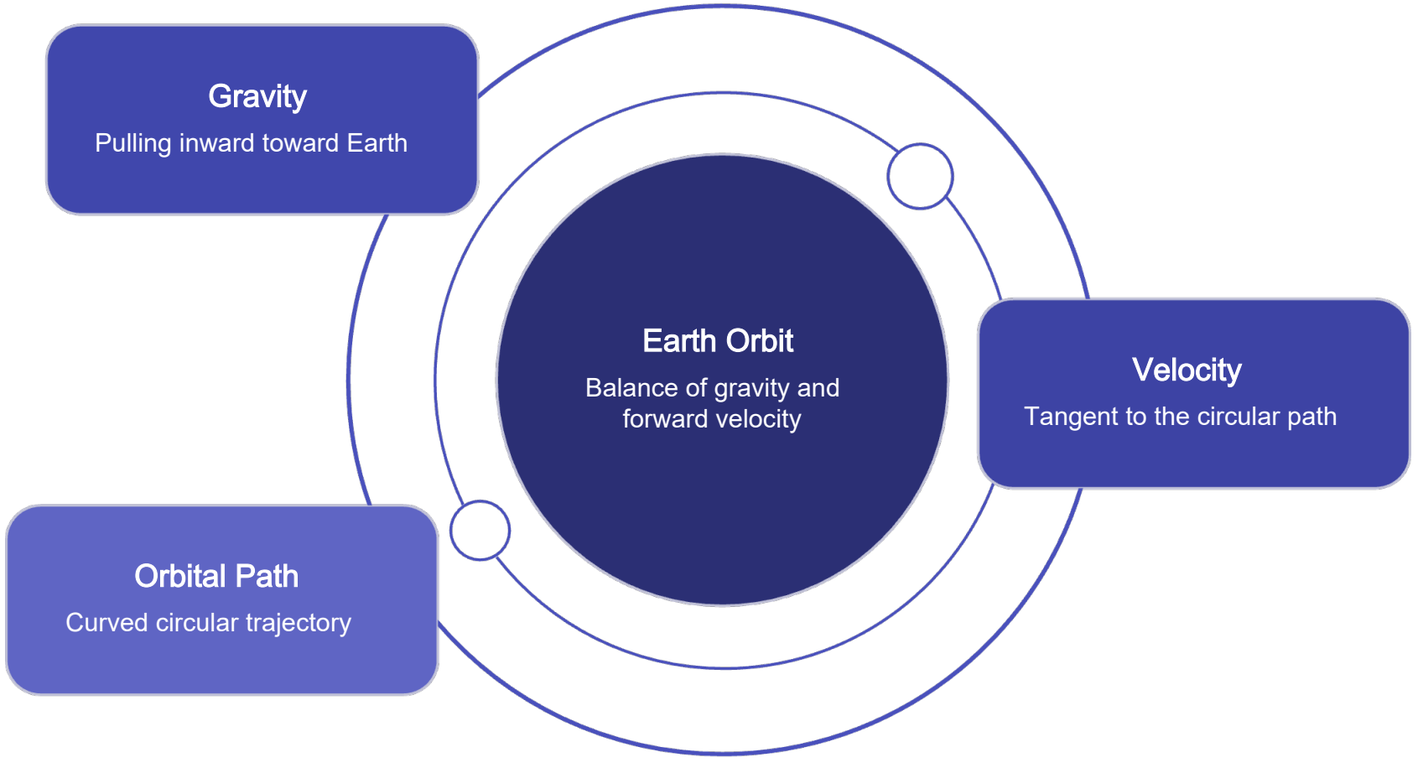
DAY 1

Essential Question

How do objects stay in orbit without flying away or crashing into Earth?

Aerospace engineering bridges the gap between atmospheric flight and space exploration. Understanding orbital mechanics is fundamental to designing satellites, space stations, and interplanetary missions.

What Is an Orbit?



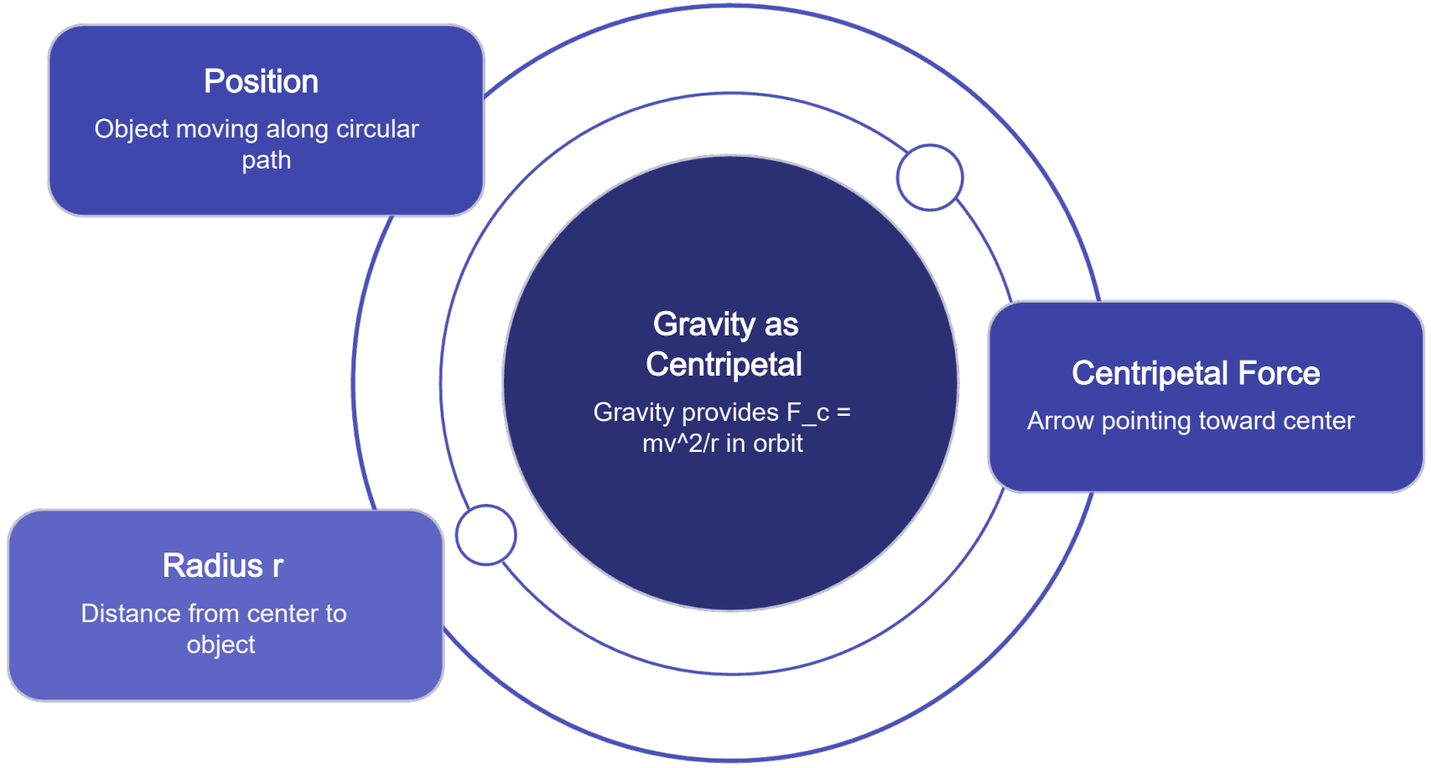
An orbit is continuous free fall around a planet. It's a delicate balance between two forces working together.

The orbital dance requires:

- **Gravity** pulling the object inward toward Earth's center
- **Forward velocity** carrying the object sideways fast enough to keep missing the planet

Without gravity, the object would fly off into space. Without velocity, it would crash straight down.

Gravity as Centripetal Force



The Physics of Circular Motion

Any object moving in a circle needs a force pulling it toward the center. This is called **centripetal force** .

$$F_c = \frac{mv^2}{r}$$

In orbit, **gravity provides this centripetal force** . The satellite wants to move in a straight line, but gravity constantly pulls it into a curved path.

- **m** = mass of the orbiting object
- **v** = velocity (speed)
- **r** = orbital radius

Newton's Law of Gravitation

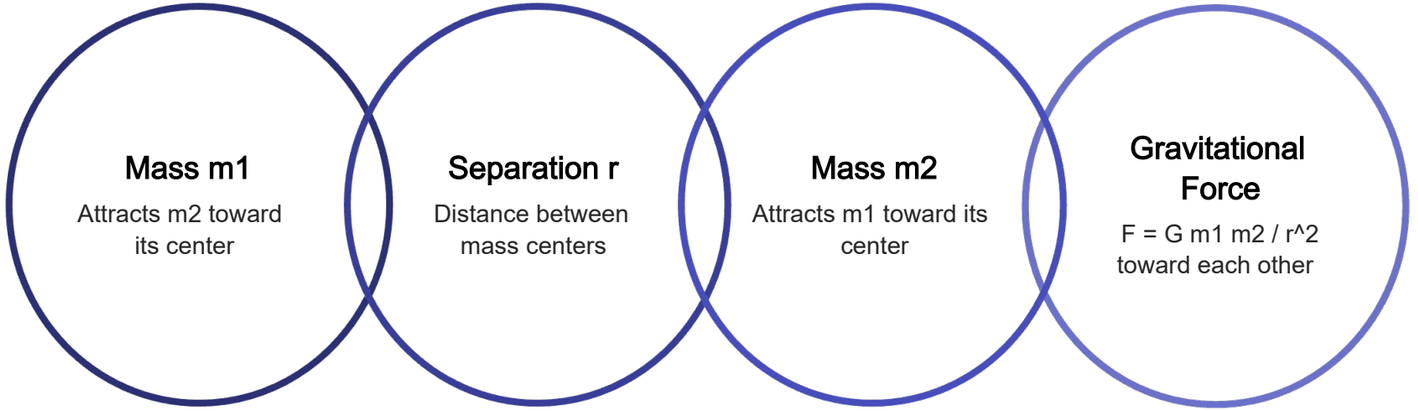
The Universal Force

Every object with mass attracts every other object with mass. Newton discovered the mathematical relationship:

$$F = G\frac{m_1 m_2}{r^2}$$

where **G** is the gravitational constant.

Key insight: If distance doubles, gravitational force becomes 1/4 as strong. This inverse-square relationship is crucial for understanding orbital mechanics.



Universal Gravitational Constant (G)

G = 6.674 \times 10^{-11}\text{ N} \cdot \text{m}^2/\text{kg}^2

A fundamental constant of nature that determines the strength of gravitational attraction.

It tells us how strong gravity is in the universe.

Important:

- G is the same everywhere.
- It does not change from planet to planet.

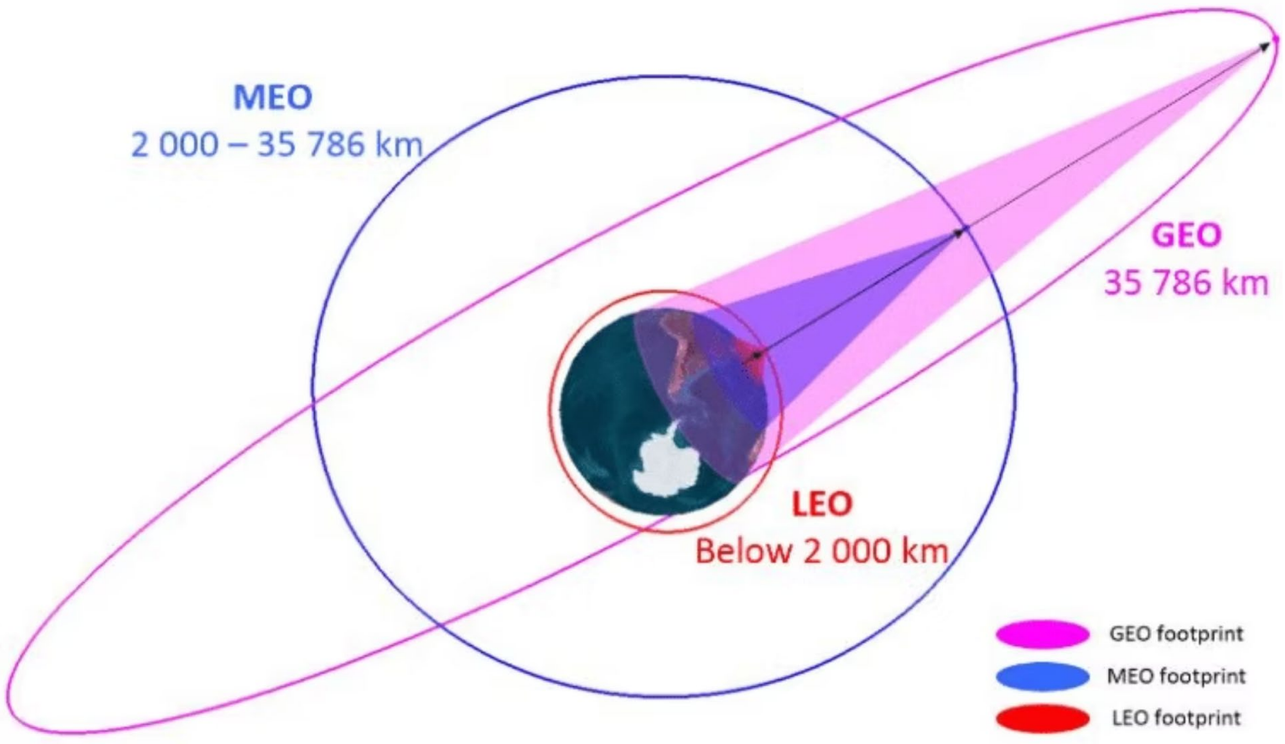
Because G is very small, gravity between everyday objects is weak.

Why Astronauts Are "Weightless"

Astronauts aboard the International Space Station aren't escaping gravity —they're in continuous free fall! The ISS and everything inside it are constantly falling toward Earth, but moving forward fast enough to keep missing it.

Surprising fact: At the ISS altitude of ~400 km, gravity is still about 90% as strong as on Earth's surface. Astronauts feel weightless because there's no floor pushing up against them —they're falling at the same rate as the station.

Types of Orbits



LEO

Low Earth Orbit

160-2,000 km altitude

ISS, Starlink, imaging satellites

MEO

Medium Earth Orbit

2,000 -35,786 km

GPS satellites, navigation systems

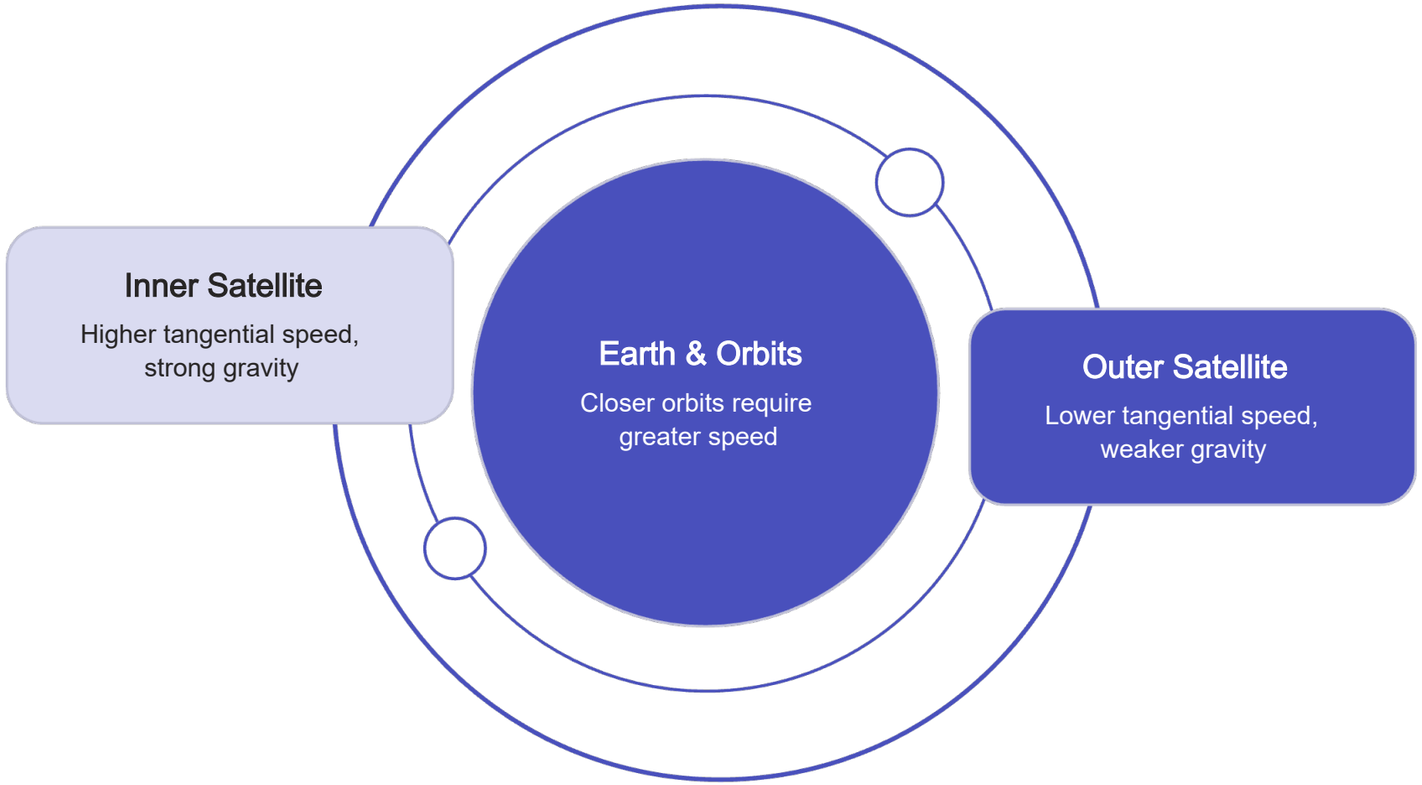
GEO

Geostationary Orbit

35,786 km altitude

Weather satellites, communications

Why Lower Orbits Move Faster



Orbital velocity depends on distance from Earth:

$$v = \sqrt{\frac{GM}{r}}$$

Notice that velocity is **inversely proportional** to the square root of radius.

What this means: Satellites closer to Earth must move faster to maintain orbit. The ISS travels at ~7.7 km/s, while geostationary satellites move at only ~3.1 km/s.

Smaller r → larger velocity

Deriving the Orbital Velocity Equation

Let's combine Newton's Law of Gravitation with the centripetal force equation to find orbital velocity.

1

Start with force balance

Gravitational force = Centripetal force needed

$$G\frac{mM}{r^2} = \frac{mv^2}{r}$$

2

Cancel mass m

Notice the orbiting mass m appears on both sides

$$G\frac{M}{r^2} = \frac{v^2}{r}$$

3

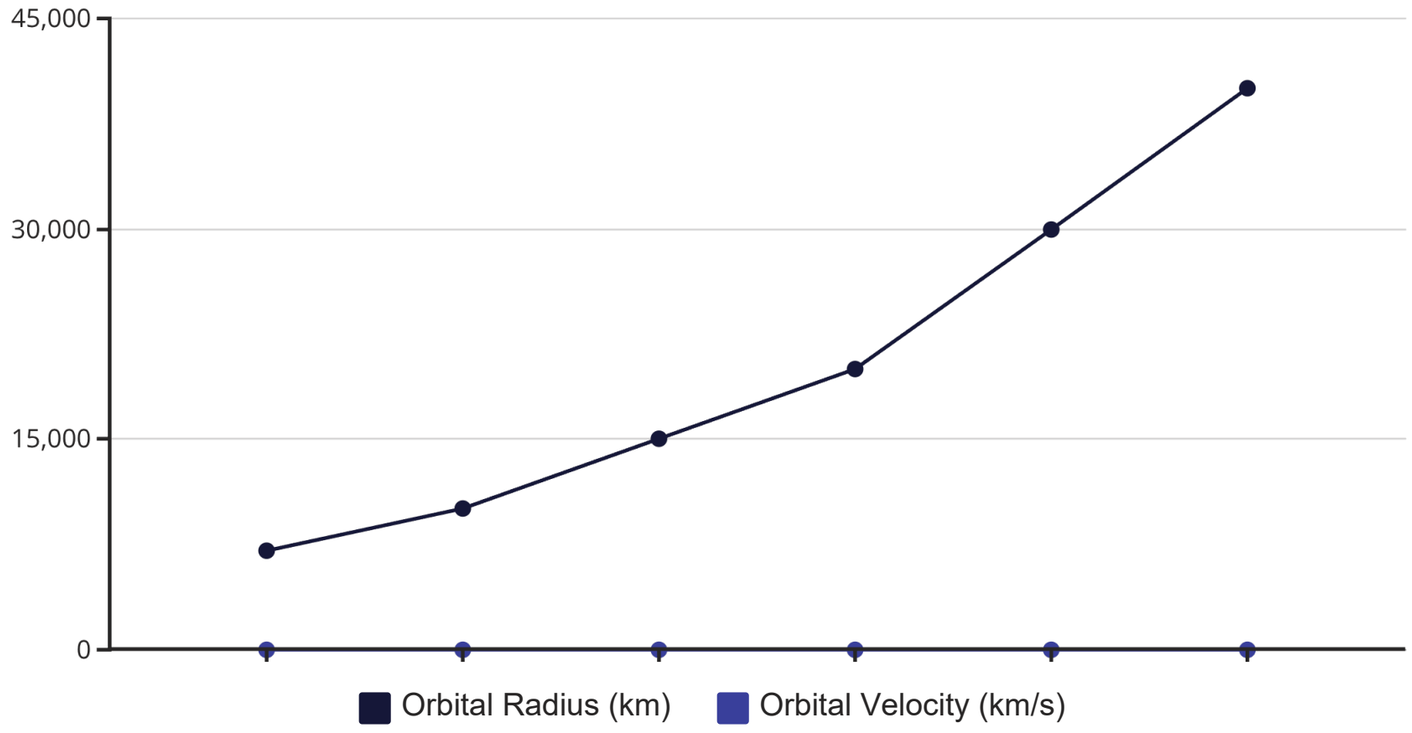
Solve for velocity

Multiply both sides by r and take the square root

$$v = \sqrt{\frac{GM}{r}}$$

This elegant result shows that orbital velocity depends only on the planet's mass (M) and the orbital radius (r) —not on the satellite's mass!

Velocity vs Radius Relationship



Understanding the Curve

The graph shows the **inverse square root relationship** between orbital radius and velocity:

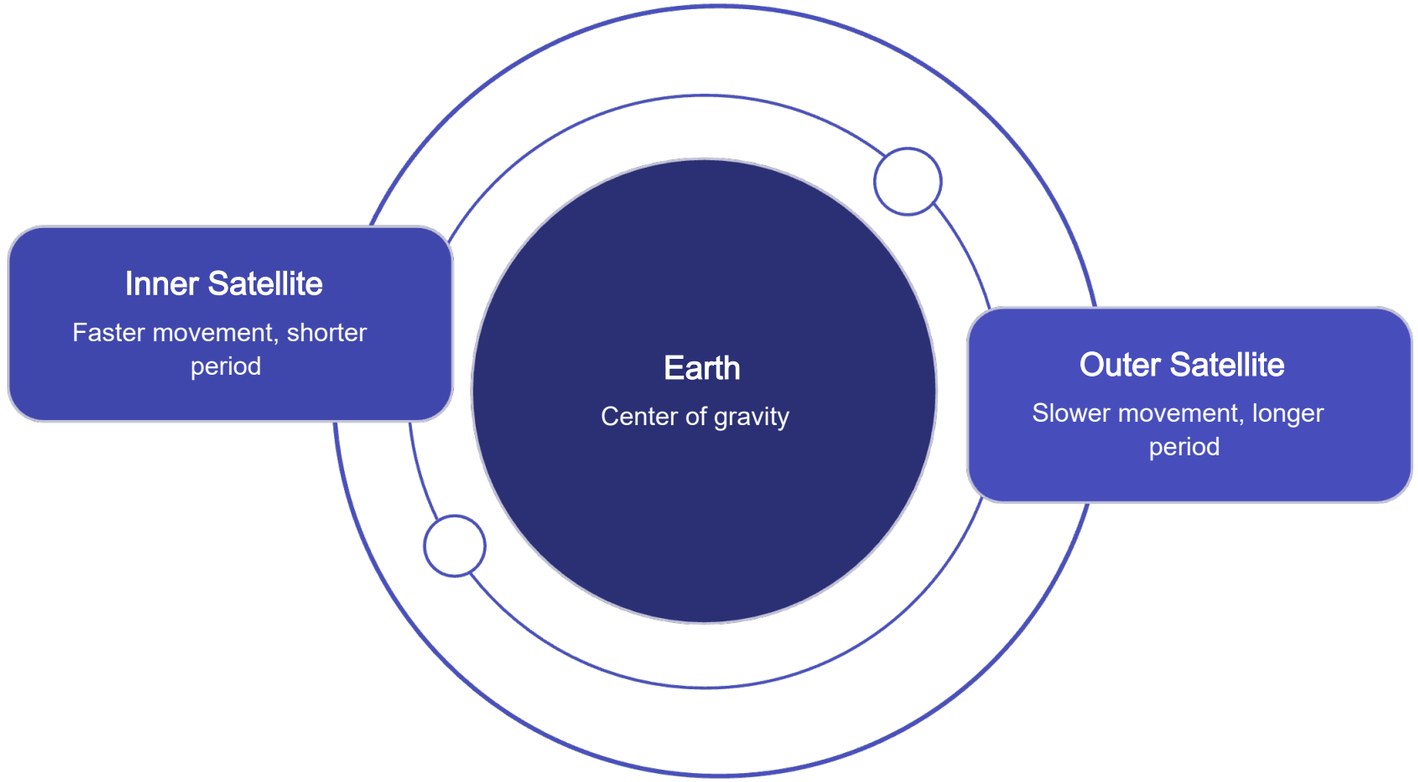
$$v = \sqrt{\frac{GM}{r}}$$

Key observations:

- The curve slopes downward but is **not linear**
- Velocity decreases as radius increases
- The relationship follows a smooth curve, not a straight line

This reinforces proportional reasoning: doubling the radius doesn't halve the velocity —it reduces it by a factor of $\sqrt{2} \approx 0.707$.

Orbital Period



Time to Complete One Orbit

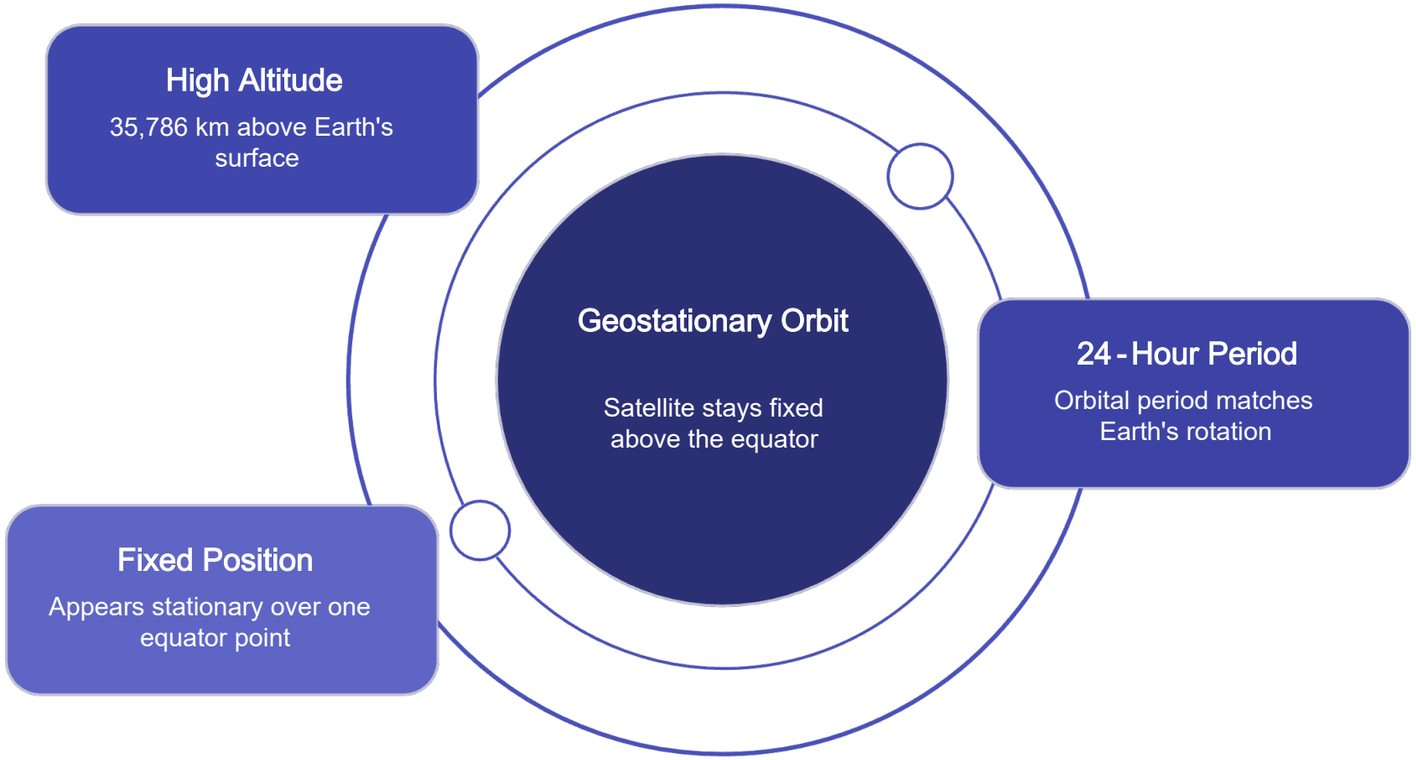
Orbital Period = Time required to complete one full orbit.

Key relationship:

- Lower orbit → higher velocity → shorter period
- Higher orbit → lower velocity → longer period

Even though satellites in higher orbits move slower, they travel a much larger circular path.

What Is a Geostationary Orbit?



Staying Fixed Above Earth

A Geostationary Orbit (GEO):

- Altitude: 35,786 km above Earth
- Orbital period: 24 hours
- Appears fixed over one location on Earth

Why it works:

The satellite's orbital period matches Earth's rotation.

Why GEO Must Be So High

The Key Question

Why can't a satellite in Low Earth Orbit stay over one location?

Understanding the Physics

LEO satellites move too fast.

To match Earth's 24 -hour rotation, a satellite must:

- Move more slowly
- Have a longer orbital period
- Be at a larger orbital radius

From the equation:

$$v = \sqrt{\frac{GM}{r}}$$

If r increases → velocity decreases.

This slower velocity produces a longer orbital period.

Why GEO Must Be So High

This diagram illustrates that GEO's higher altitude results in slower orbital velocity, allowing it to synchronize with Earth's rotation and appear fixed, unlike fast -moving LEO satellites.

The Key Question

Why can't a LEO satellite stay over one location?

Understanding the Physics

LEO satellites move too fast to appear fixed. To match Earth's 24-hour rotation, a satellite must:

- Move more slowly
- Have a longer orbital period
- Be at a larger orbital radius

From the orbital velocity equation : $v = \sqrt{\frac{GM}{r}}$

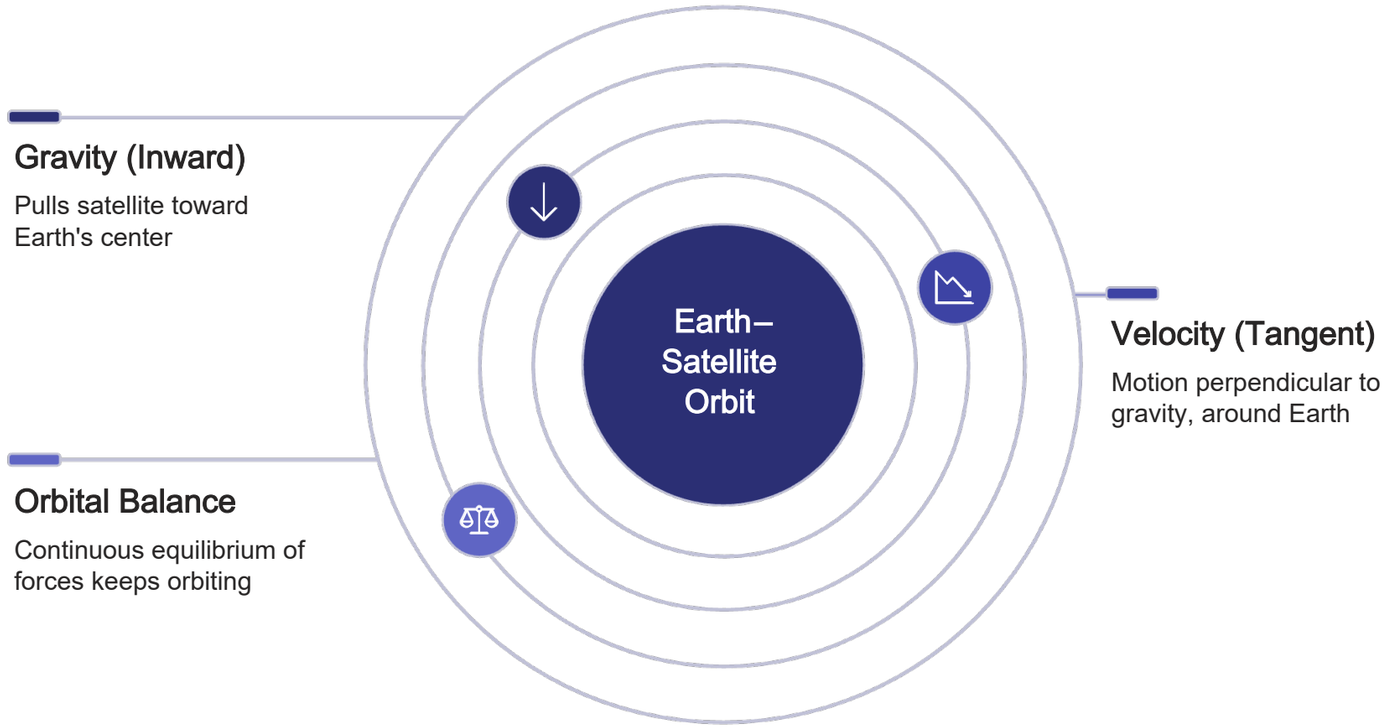
An increase in r (orbital radius) leads to a decrease in velocity. This slower velocity at a greater distance provides the necessary 24 -hour orbital period for a geostationary satellite.

Conceptual Check: The Orbital Balance

Understanding the Big Picture

Let's reinforce the fundamental concept: an orbit is a continuous balancing act between two perpendicular components.

Remove either component and the orbit fails: too slow and the object crashes; too fast and it escapes; eliminate gravity and it flies off tangentially.



Gravity (Inward)

Pulls the object toward Earth's center, preventing it from flying off into space

Velocity (Sideways)

Carries the object forward fast enough to keep "missing" Earth as it falls

Practice Problem

Challenge Question: If the orbital radius of a satellite doubles, what happens to its velocity?

01

Write the equation

Start with $v = \sqrt{GM/r}$

02

Substitute 2r for r

New velocity: $v_{new} = \sqrt{GM/(2r)}$

03

Simplify

$v_{new} = \sqrt{\frac{1}{2}} \cdot \sqrt{GM/r} = v/\sqrt{2}$

04

Interpret the result

Velocity decreases by a factor of $\frac{1}{\sqrt{2}} \approx 0.707$, or about 70.7% of the original velocity

Conceptual Check: The Orbital Balance

Understanding the Big Picture

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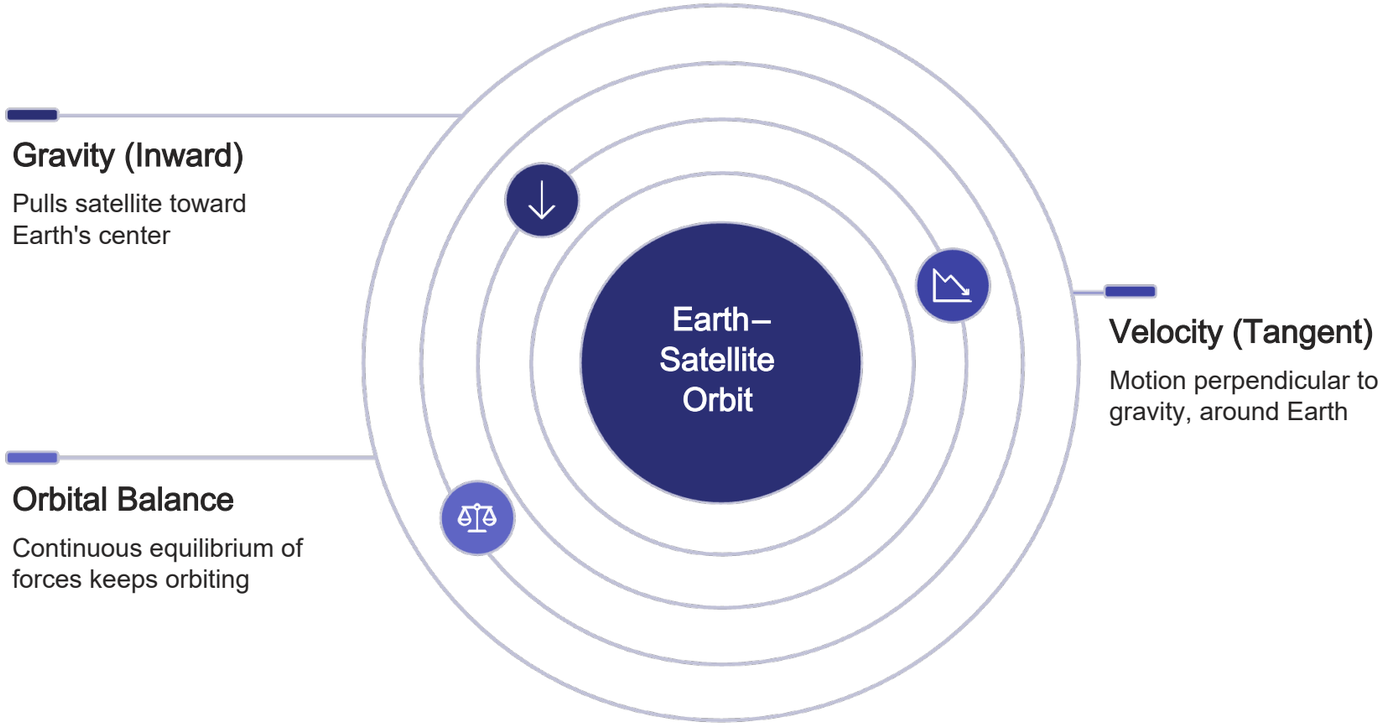

Gravity (Inward)

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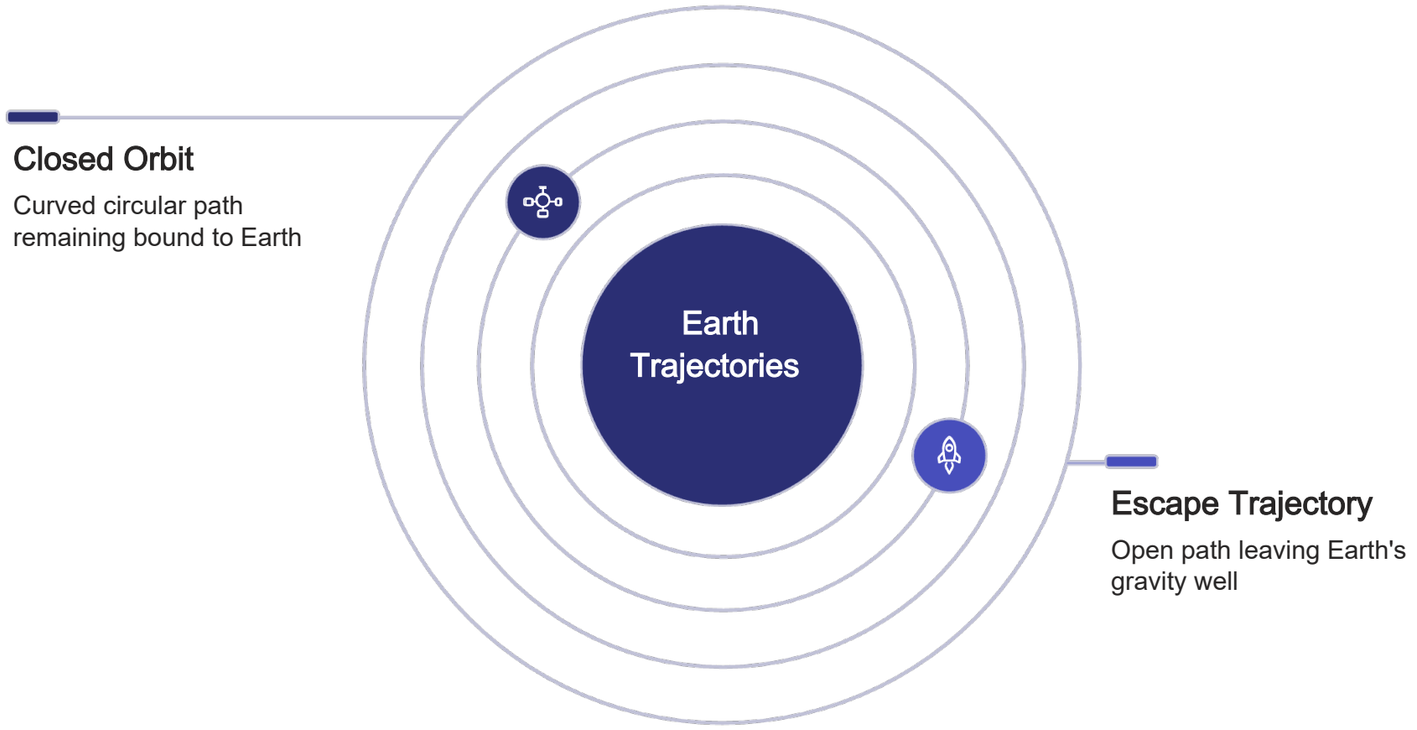
Escape Velocity



Breaking Free from Earth's Gravity

To completely escape Earth's gravitational pull, an object must reach **escape velocity** :

$$v_{escape} \approx 11.2 \text{ km/s}$$

This is about 40,320 km/h (25,020 mph) — incredibly fast!

The key difference:

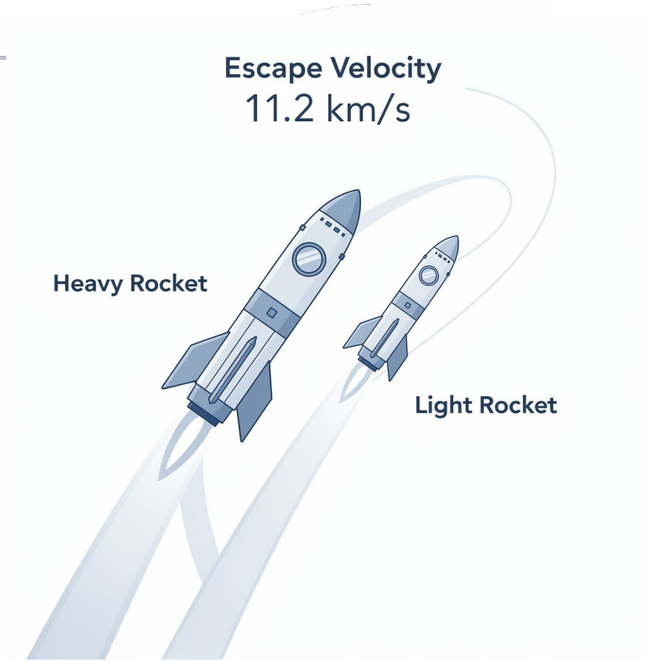
- **Orbital velocity:** Circular path, stays bound to Earth
- **Escape velocity:** Open trajectory, leaves Earth's influence forever

Escape velocity requires significantly more energy than achieving orbit.

Escape Velocity: Mass Doesn't Matter

Heavy Rocket

Large mass, same required 11.2 km/s to escape Earth.



Light Rocket

Small mass, still needs 11.2 km/s to escape Earth.

Breaking a Common Misconception

A surprising result: escape velocity is the **same for all objects**, regardless of their mass!

$$v_{escape} = \sqrt{\frac{2GM}{r}}$$

Notice there's no mass term for the escaping object. Whether you're launching a tennis ball or a massive rocket, both need the same velocity to escape Earth's gravity.

Why this matters: While the required *velocity* is the same, the *energy* needed (which depends on mass) is vastly different. A heavier rocket requires much more fuel to reach escape velocity.

Homework & Key Takeaways

Review Today's Concepts

- Orbits are continuous free fall with balanced forces
- Gravity provides centripetal force for circular motion
- Lower orbits require higher velocities
- Escape velocity doesn't depend on object mass

Practice Problems

1. Calculate orbital velocity for a satellite at 10,000 km altitude
2. Explain why astronauts feel weightless in the ISS
3. Compare the velocities needed for LEO vs GEO orbits
4. Derive the relationship between orbital period and radius

Looking Ahead

Next class: We'll explore orbital transfers, the Hohmann transfer orbit, and how spacecraft change their orbits efficiently. Bring your calculators!