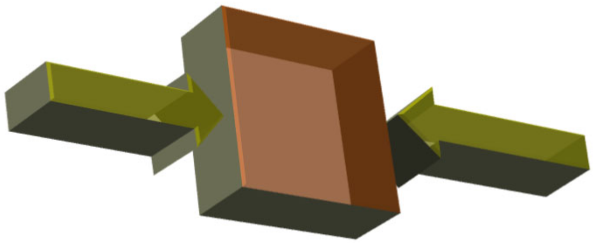
Calculating Truss Forces

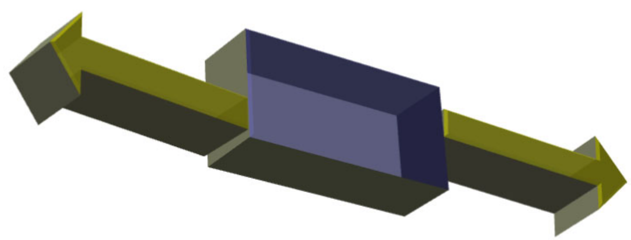
Forces

Compression



A body being squeezed

Tension



A body being stretched

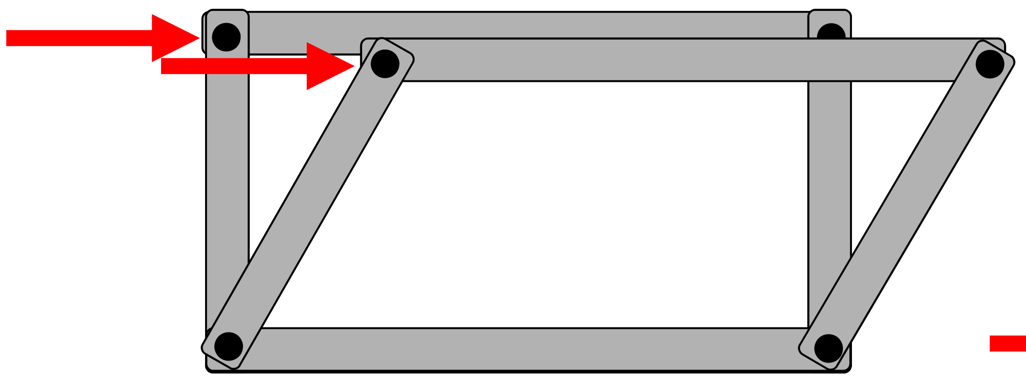
Truss

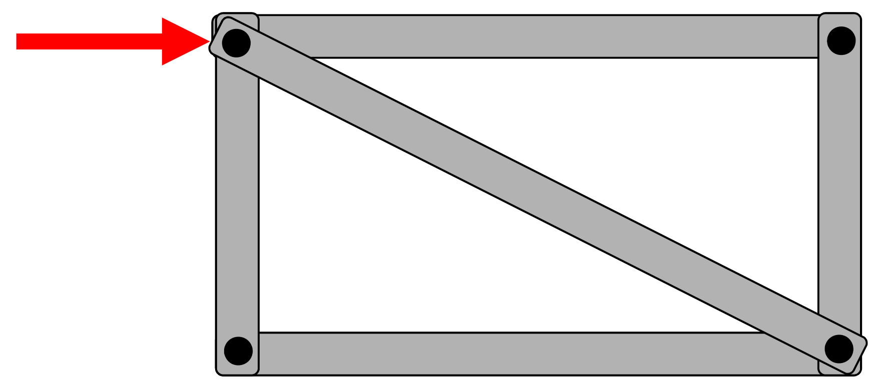
A truss is composed of slender members joined together at their end points.

- They are usually joined by welds or gusset plates.

Simple Truss

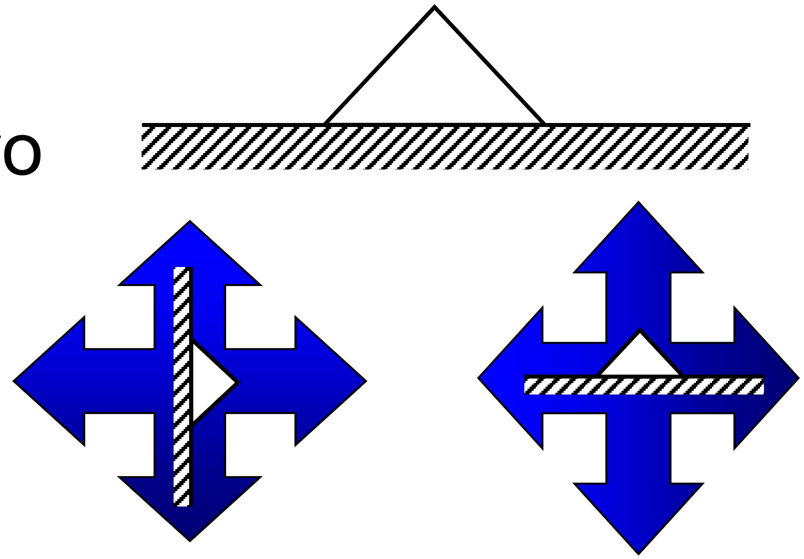
A simple truss is composed of triangles, which will retain their shape even when removed from supports.

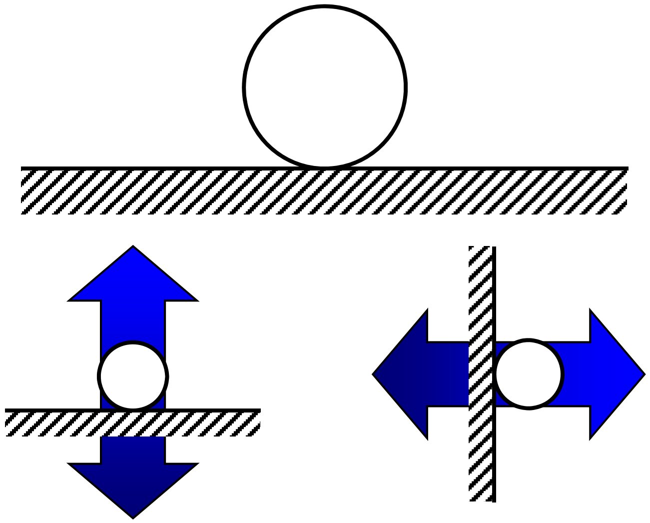




Pinned and Roller Supports

A **pinned** support can support a structure in two dimensions.





A **roller** support can support a structure in only one dimension.

Solving Truss Forces

Assumptions:

All members are perfectly straight.

All loads are applied at the joints.

All joints are pinned and frictionless.

Each member has no weight.

Members can only experience tension or compression forces.

What risks might these assumptions pose if we were designing an actual bridge?

Static Determinacy

A statically determinate structure is one that can be mathematically solved.

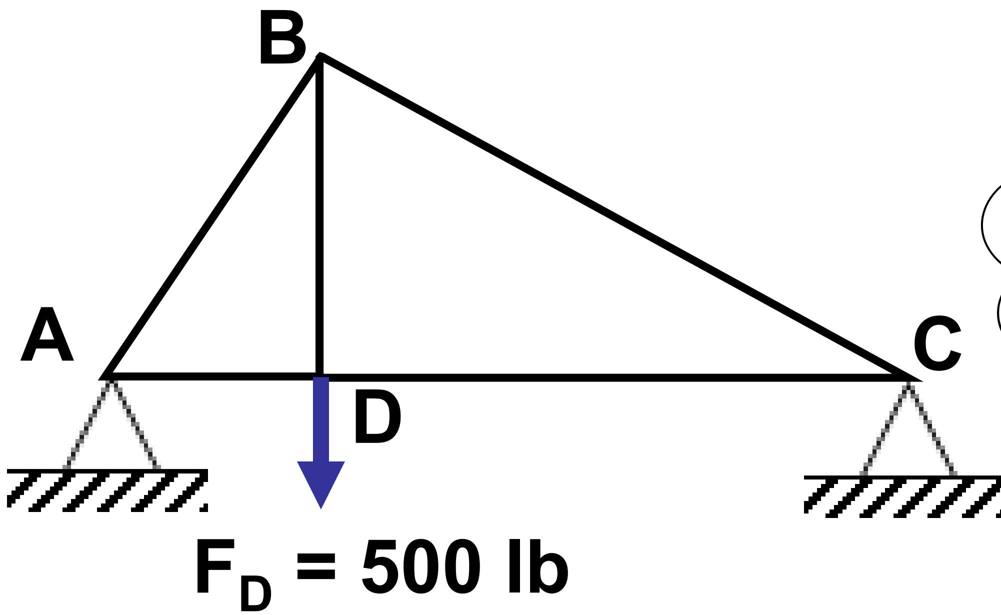
$$2J = M + R$$

J = Number of Joints

M = Number of Members

R = Number of Reactions

Statically Indeterminate



Each pin connection contributes TWO reaction forces

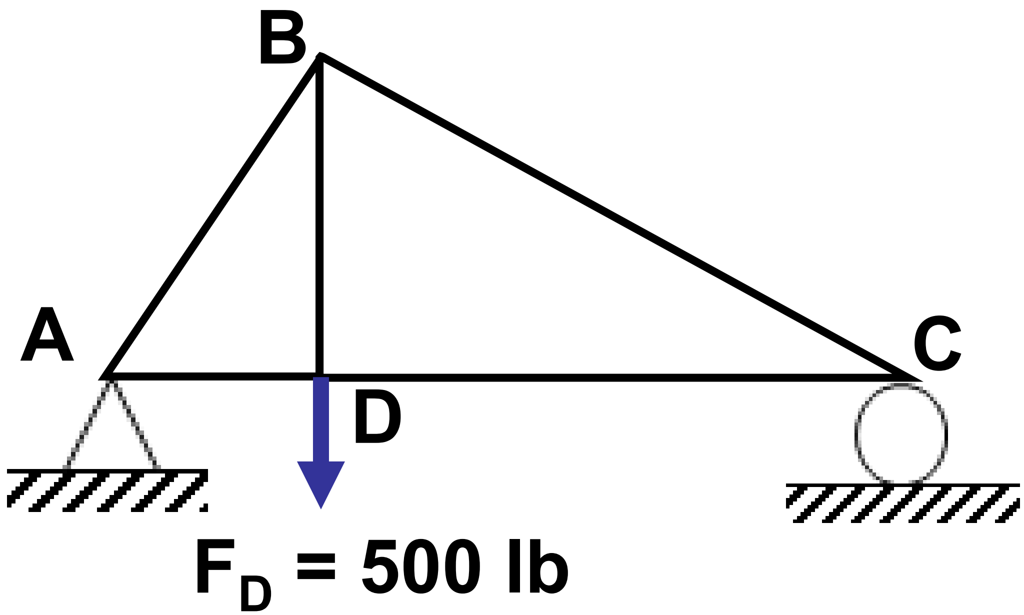
A truss is considered **statically indeterminate** when the static equilibrium equations are not sufficient to find the reactions on that structure. There are simply too many unknowns.

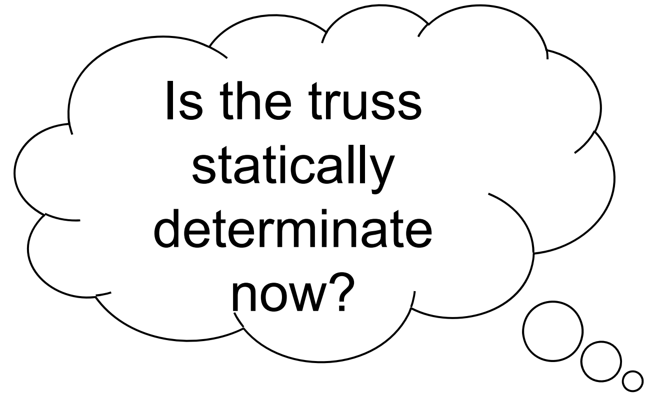
Try It

$$2J = M + R$$

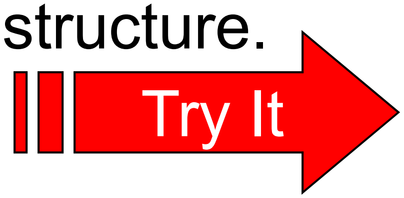
$$2(4) \neq 5 + 4$$

Statically Determinate





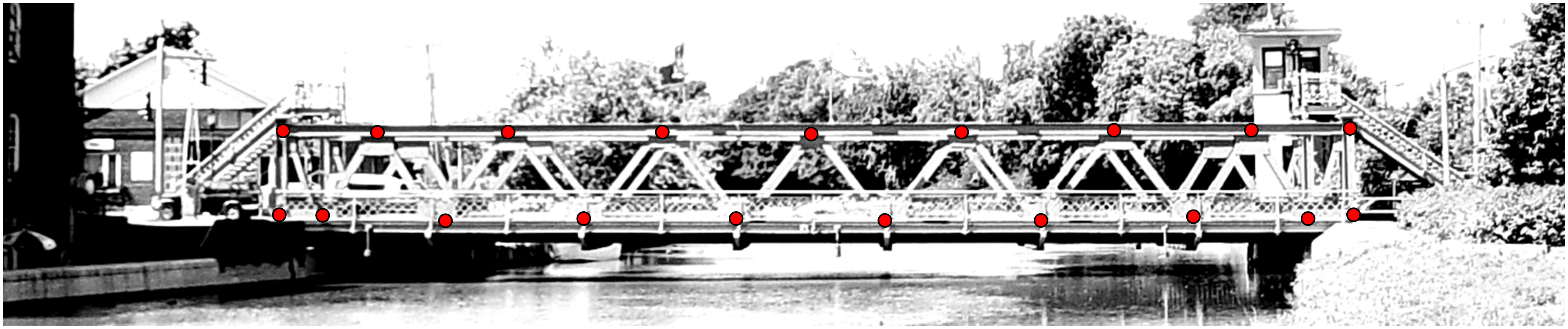
A truss is considered **statically determinate** when the static equilibrium equations can be used to find the reactions on that structure.



$$2J = M + R$$

$$2(4) = 5 + 3$$

Static Determinacy Example

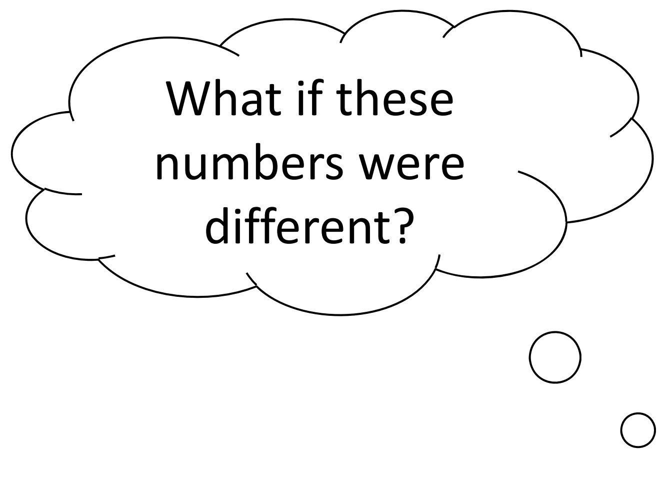


Each side of the main street bridge in Brockport, NY has 19 joints, 35 members, and three reaction forces (pin and roller), making it a statically determinate truss.

$$2J = M + R$$

$$2(19) = 35 + 3$$

$$38 = 38$$



Equilibrium Equations

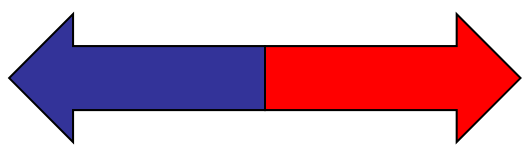
$$\Sigma M = 0$$

The sum of the moments about a given point is zero.

Equilibrium Equations

$$\Sigma F_x = 0$$

The sum of the forces in the x-direction is zero.

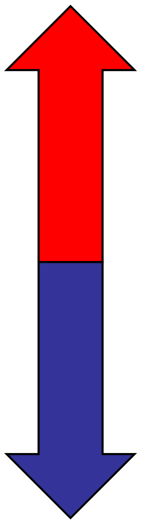


Do you remember the Cartesian coordinate system? A vector that acts to the right is positive, and a vector that acts to the left is negative.

Equilibrium Equations

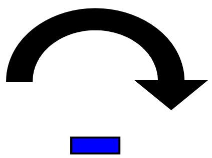
$$\Sigma F_y = 0$$

The sum of the forces in the y-direction is zero.



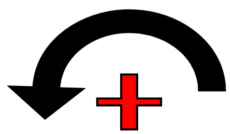
A vector that acts up is positive, and a vector that acts down is negative.

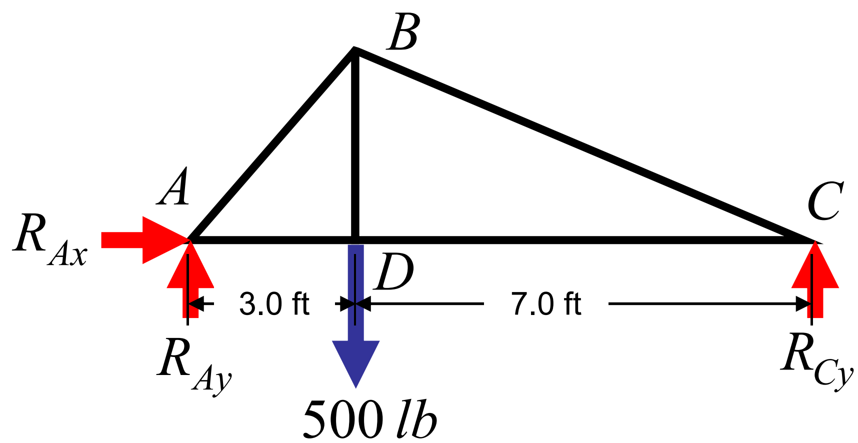
Using Moments to Find R_{CY}



A force that causes a **clockwise** moment is a **negative moment.**

A force that causes a **counterclockwise** moment is **positive moment.**





$$\sum M_A = 0$$

$$-F_D(3.0\ \text{ft}) + R_{C_y}\,(10.0\ \text{ft}) = 0$$

$$-500\ \text{lb}\,(3.0\ \text{ft}) + R_{C_y}\,(10.0\ \text{ft}) = 0$$

$$-1500\ \text{lb}\cdot\text{ft} + R_{C_y}\,(10.0\ \text{ft}) = 0$$

$$R_{C_y}\,(10.0\ \cancel{\text{ft}}) = 1500\ \text{lb}\cdot\cancel{\text{ft}}$$

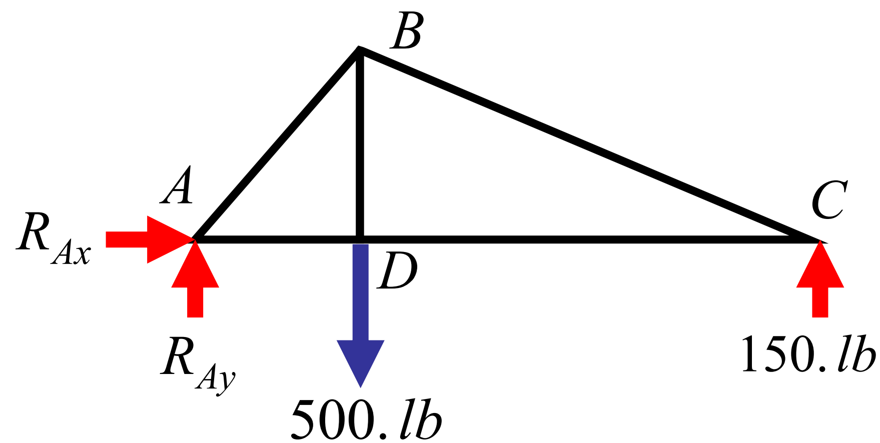
$$R_{C_y} = 150\ \text{lb}$$

F_D contributes a negative moment because it causes a clockwise moment about A.

R_{Cy} contributes a positive moment because it causes a counterclockwise moment around A.

Sum the y Forces to Find R_{A_y}

We know two out of the three forces acting in the y-direction. By simply summing those forces together, we can find the unknown reaction at point A.



$$\sum F_y = 0$$

$$-F_D + R_{C_y} + R_{A_y} = 0$$

$$-500.\ lb + 150.\underline{00}\ lb + R_{A_y} = 0$$

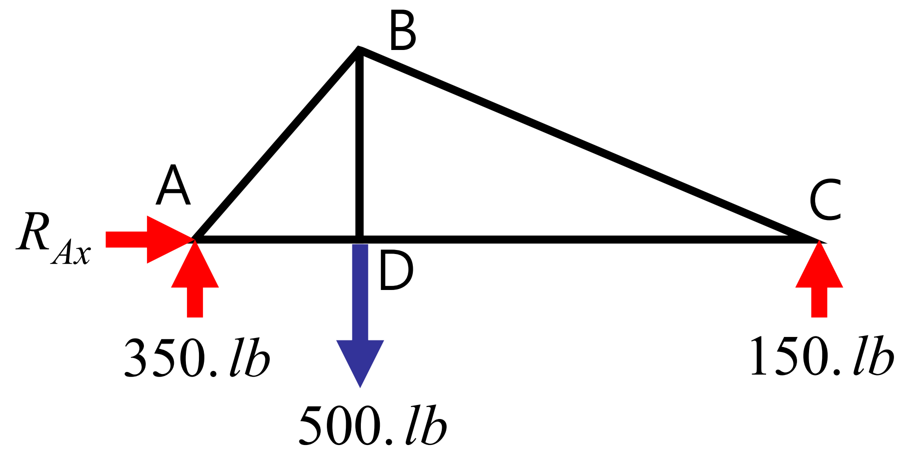
$$-350.\ lb + R_{A_y} = 0$$

$$R_{A_y} = 350.\ lb$$

Please note that a negative sign is in front of F_D because the drawing shows the force as down.

Sum the x Forces to Find A_x

Because joint **A** is pinned, it is capable of reacting to a force applied in the x-direction.



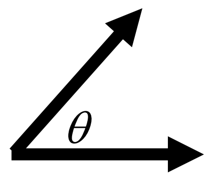
However, since the only load applied to this truss (**F_D**) has no x-component, **R_{Ax}** must be zero.

$$\sum \mathbf{F_x} = \mathbf{0}$$

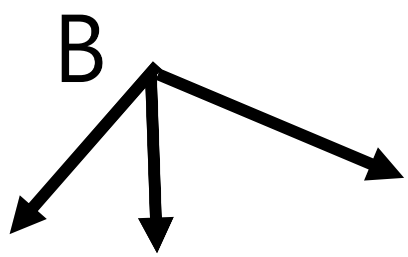
$$\mathbf{R_{A_x} = 0}$$

Method of Joints

Use cosine and sine to determine x and y vector components.



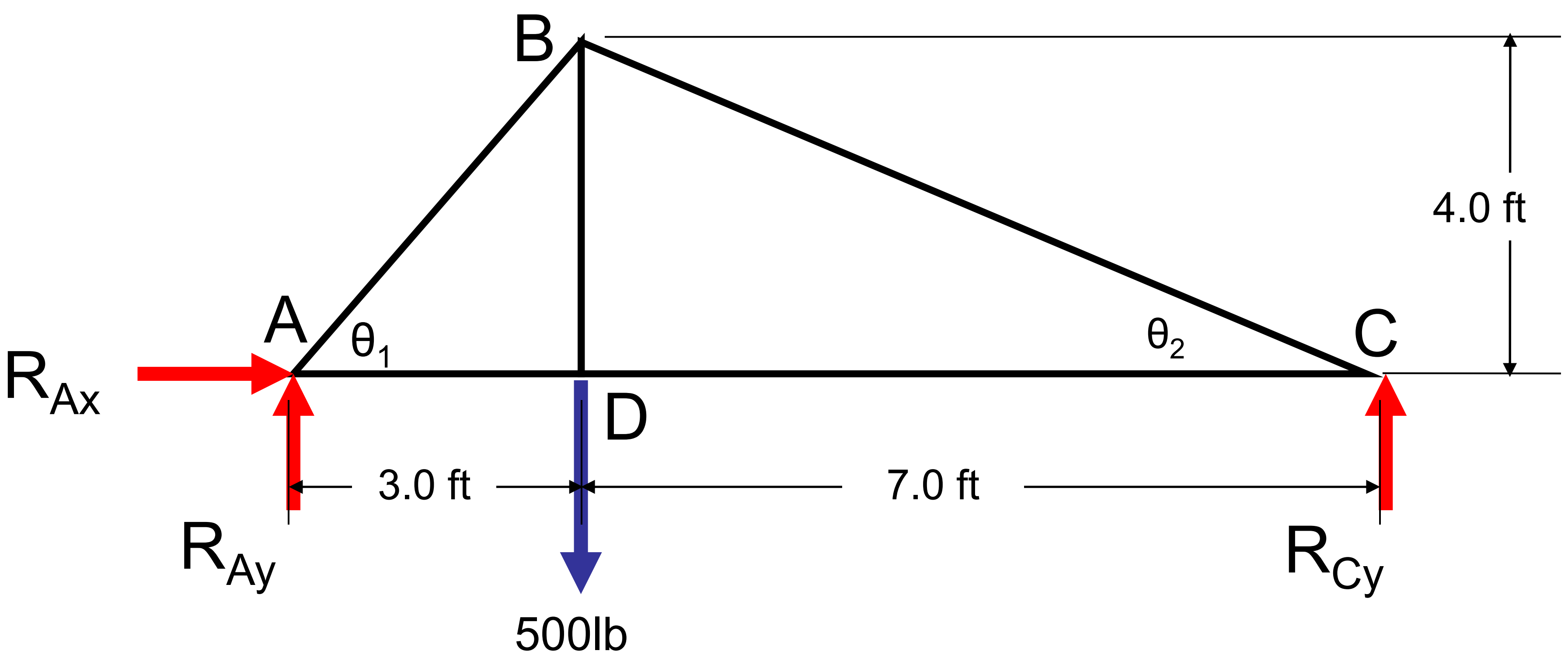
Assume all members to be in tension. A **positive** answer will mean the member is in **tension**, and a **negative** number will mean the member is in **compression**.



As forces are solved, update free body diagrams. Use correct magnitude and sense for subsequent joint free body diagrams.

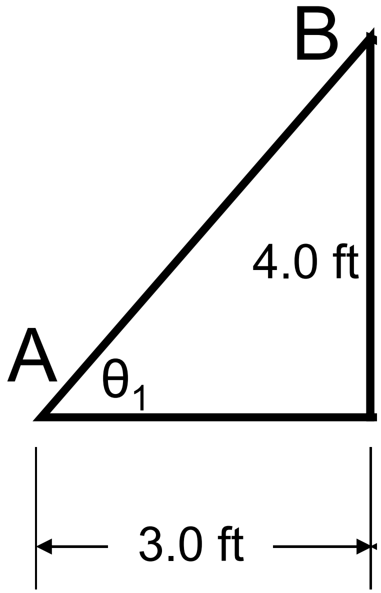
Method of Joints

Truss Dimensions



Method of Joints

Using Truss Dimensions to Find Angles



$$\tan\theta_1 = \frac{opp}{adj}$$

$$\tan\theta_1 = \frac{4.0\,\cancel{ft}}{3.0\,\cancel{ft}}$$

$$\theta_1 = \tan^{-1}\frac{4.0}{3.0}$$

$$\theta_1 = 53.130°$$

Method of Joints

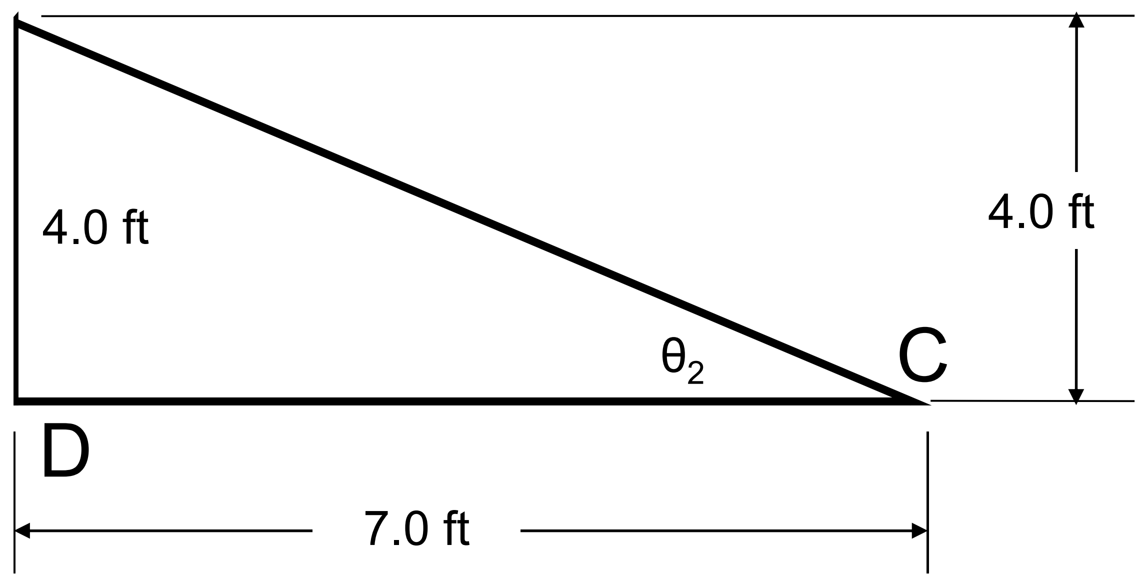
Using Truss Dimensions to Find Angles

$$\tan\theta_1 = \frac{opp}{adj}$$

$$\tan\theta_1 = \frac{4.0\,\cancel{ft}}{7.0\,\cancel{ft}}$$

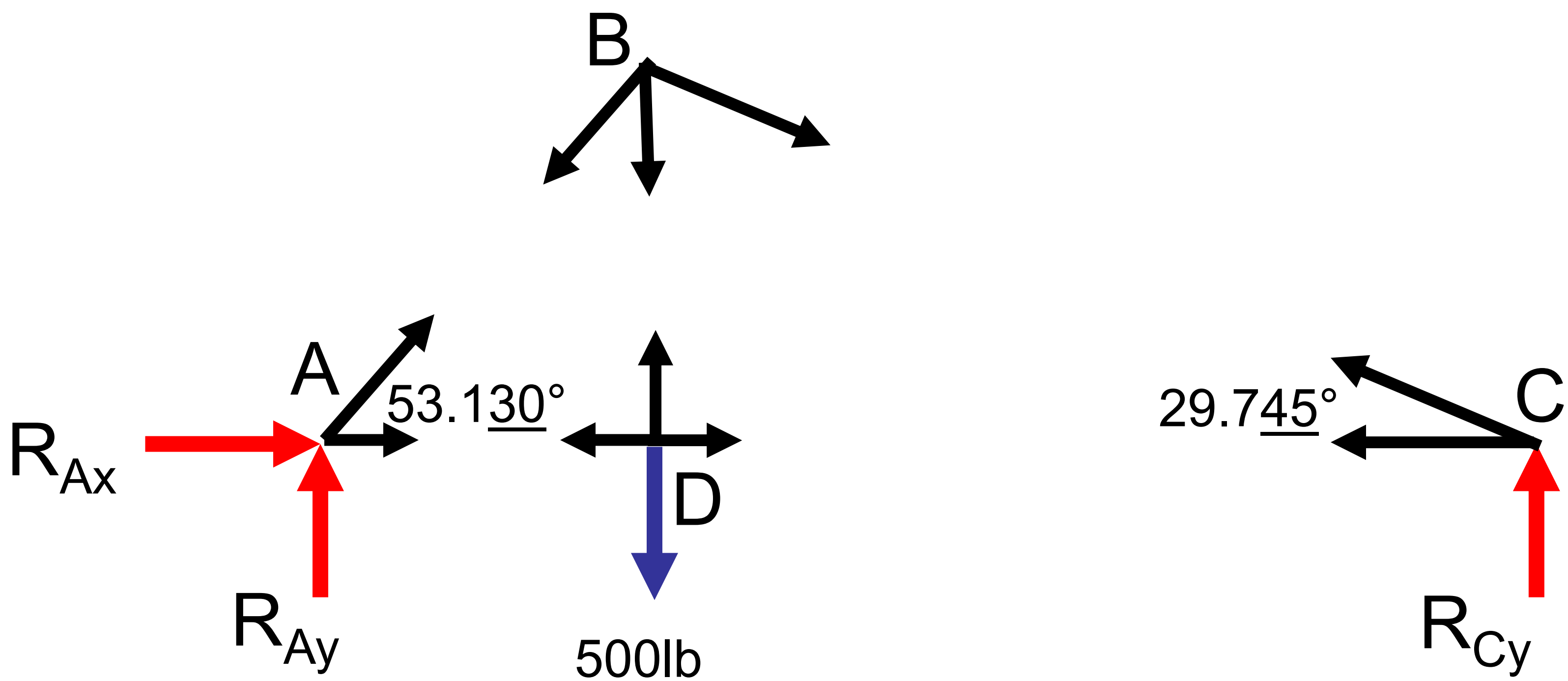
$$\theta_1 = \tan^{-1}\frac{4.0}{7.0}$$

$$\theta_1 = 29.745°$$



Method of Joints

Draw a free body diagram of each pin.



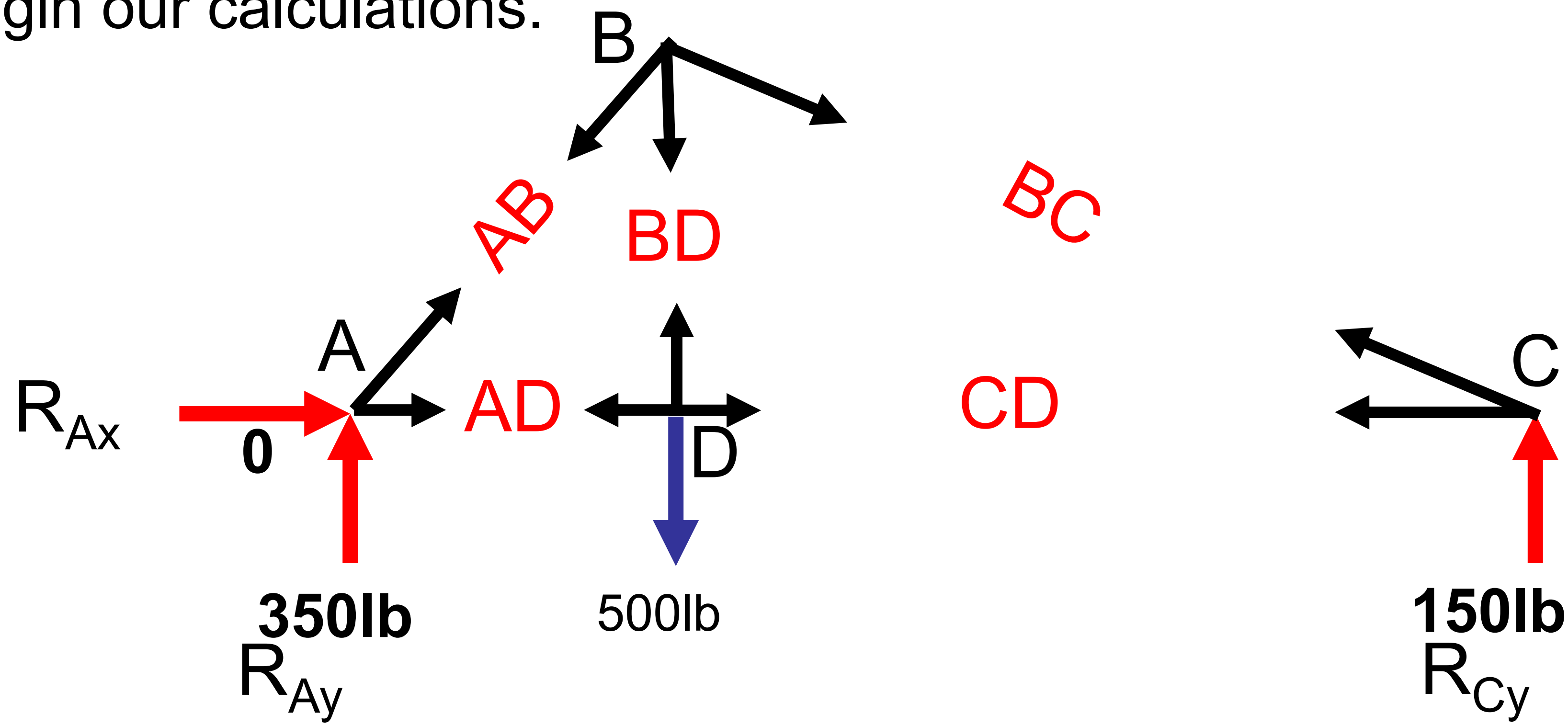
Every member is assumed to be in tension. A positive answer indicates the member is in tension, and a negative answer indicates the member is in compression.

Method of Joints

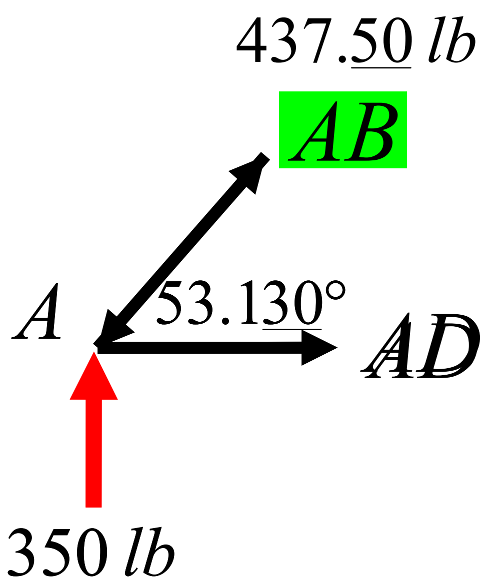
Where to Begin

Choose the joint that has the least number of unknowns.

Reaction forces at joints **A** and **C** are both good choices to begin our calculations.



Method of Joints



$$\Sigma F_Y = 0$$

$$R_{Ay} + AB_y = 0$$

$$350lb + AB\sin 53.1\underline{30}° = 0$$

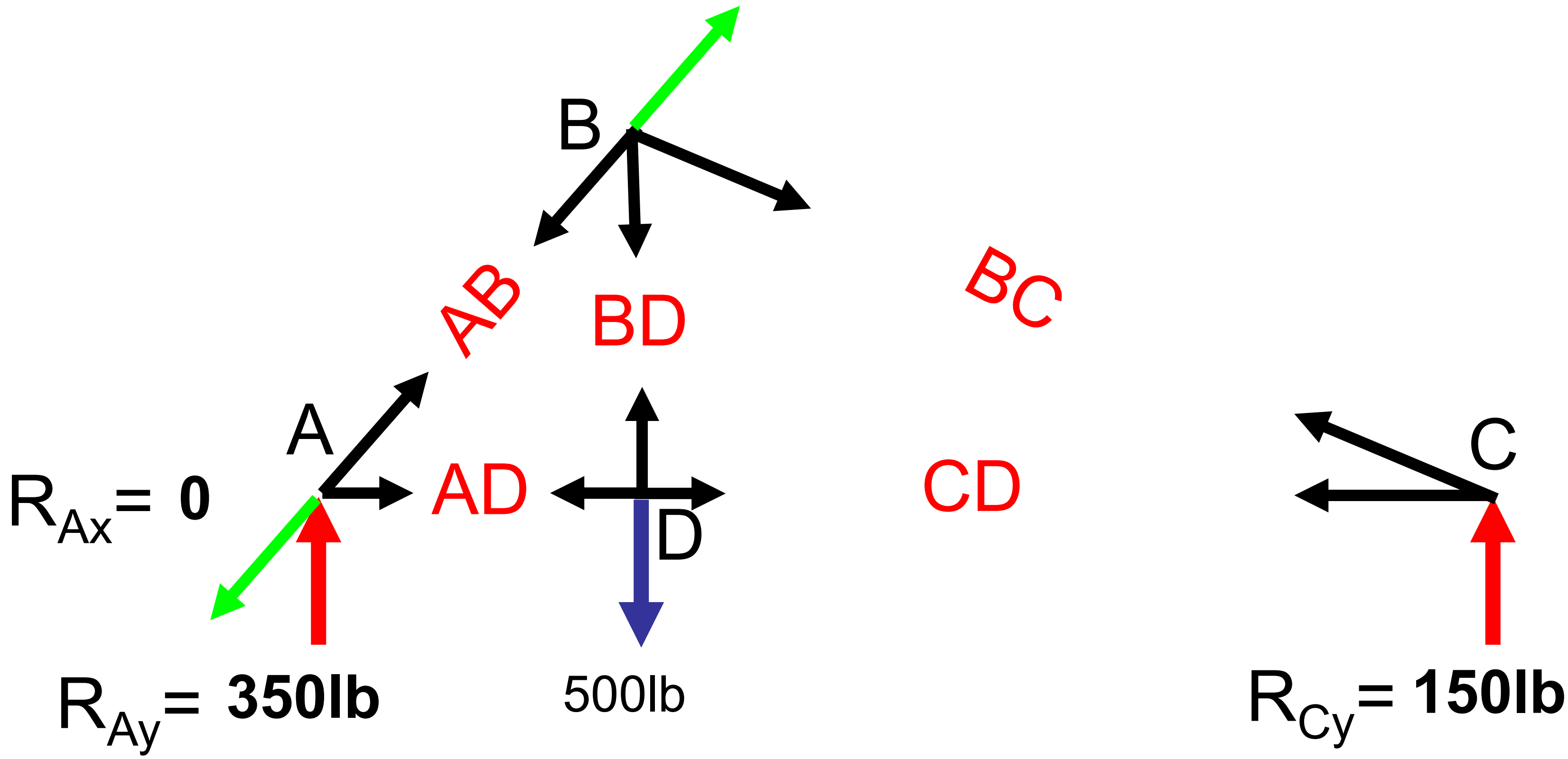
$$AB\sin 53.1\underline{30}° = -350lb$$

$$AB = \frac{-350lb}{\sin 53.1\underline{30}°}$$

$$AB = -438\ lb$$ **COMPRESSION**

Method of Joints

Update the all force diagrams based on AB being under compression.

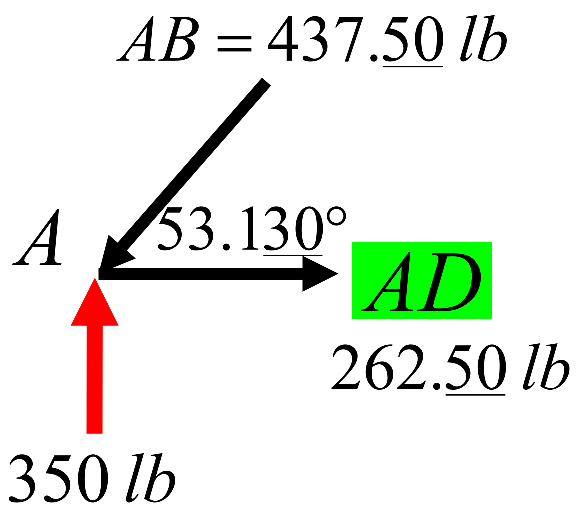


Method of Joints

$$\Sigma F_X = 0$$

$$-AB_x + AD = 0$$

$$-437.\underline{50}\ lb \cos 53.1\underline{30}° + AD = 0$$

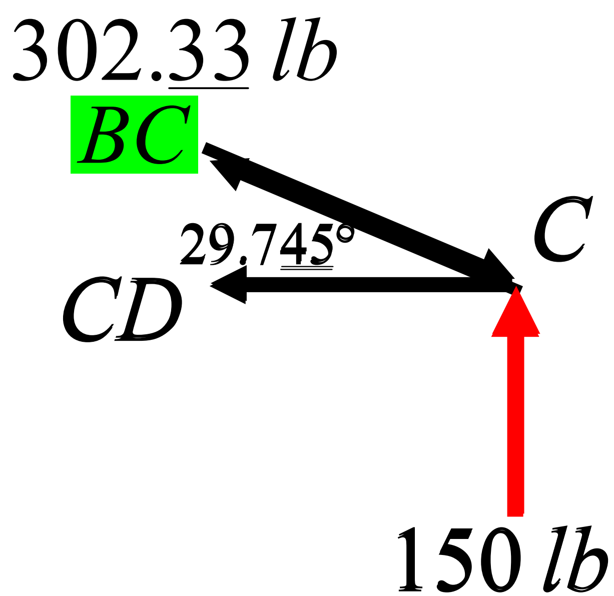


$$AD = 437.\underline{50}\ lb \cos 53.1\underline{30}°$$

$$AD = 262.50\ lb$$

TENSION

Method of Joints



$$\Sigma F_Y = 0$$

$$R_{Cy} + BC_y = 0$$

$$150\ lb + BC \sin 29.745° = 0$$

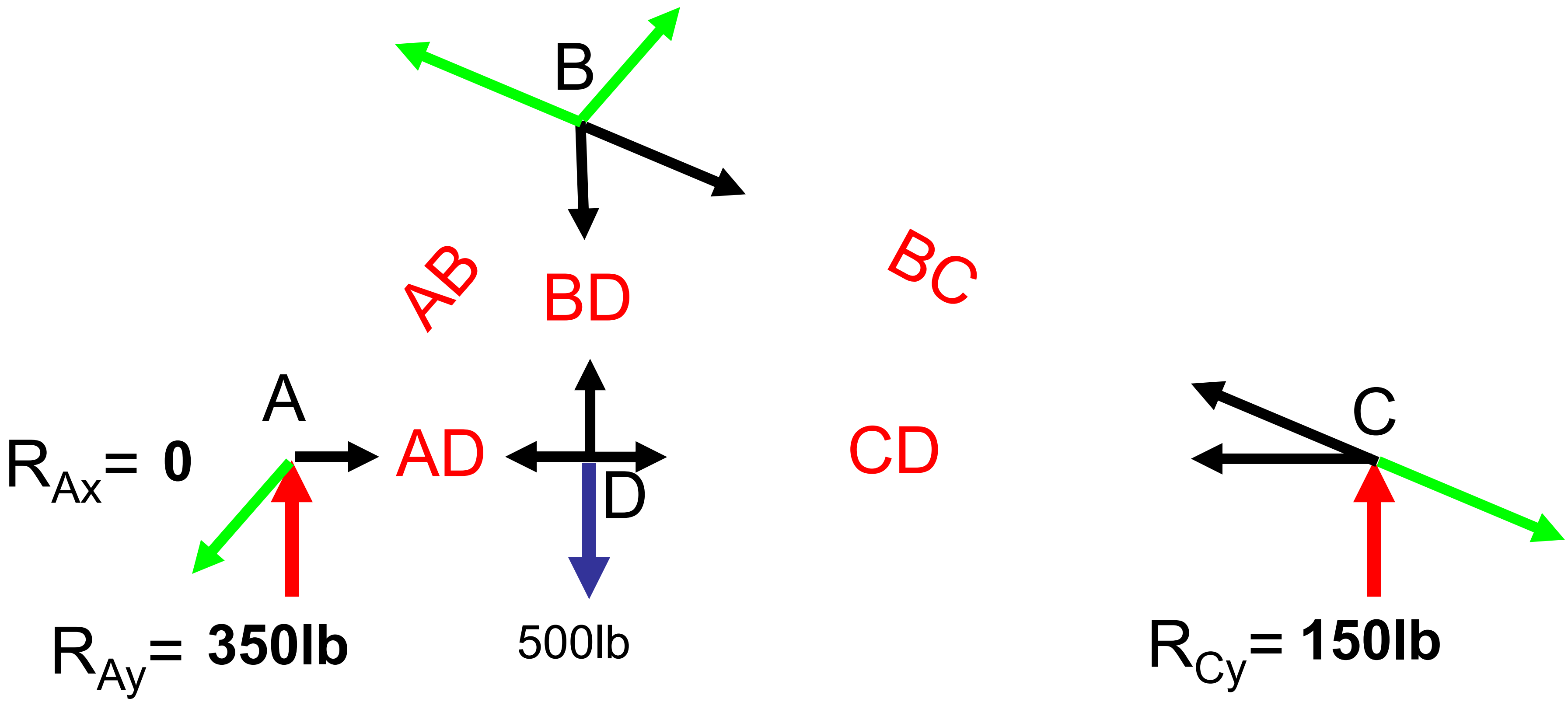
$$BC \sin 29.745° = -150\ lb$$

$$BC = \frac{-150 lb}{\sin 29.745°}$$

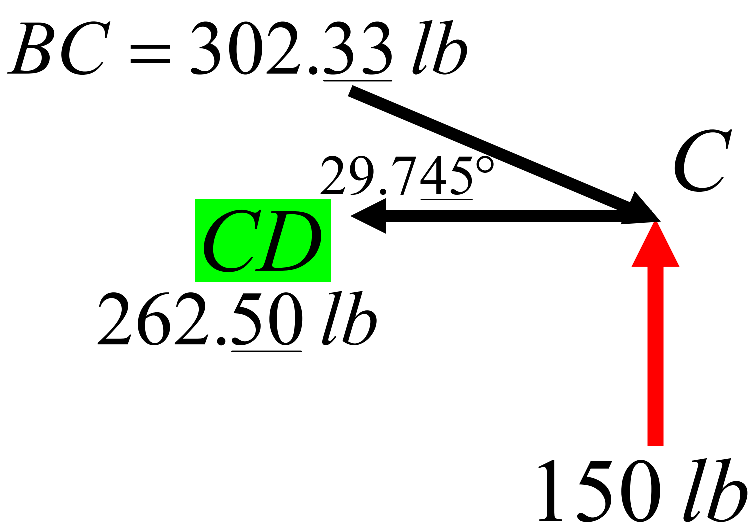
$$BC = -302\ lb$$ **COMPRESSION**

Method of Joints

Update the all force diagrams based on BC being under compression.



Method of Joints



$$\Sigma F_X = 0$$

$$BC_x - CD = 0$$

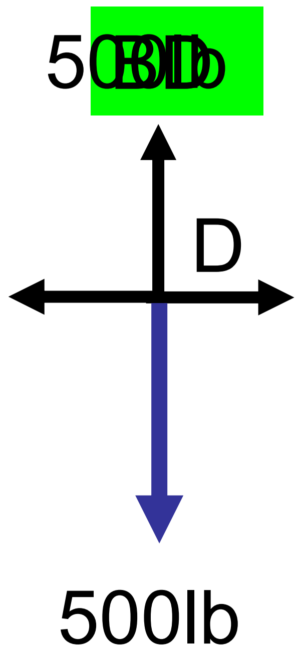
$$302.\underline{33}\ lb \cos 29.7\underline{45}° - CD = 0$$

$$CD = 302.\underline{33}\ lb \cos 29.7\underline{45}°$$

$$CD = 262.50\ lb$$

TENSION

Method of Joints



$$\Sigma F_Y = 0$$

$$BD - F_D = 0$$

$$BD - 500lb = 0$$

$$BD = 500lb$$ **TENSION**