



Experiment: Uniform Circular Motion

Created by

Richard Born

Associate Professor Emeritus

Northern Illinois University

richb@rborn.org

Topics

Physics

Math

Robotics

Ages

Junior and Senior High School Students

Duration

approx. 45 min (Junior High Students),

75 min (Senior High Students)

EXPERIMENT: UNIFORM CIRCULAR MOTION

TEACHER'S GUIDE

What will students learn?

- What is meant by the concept of *uniform circular motion*? (junior and senior high school)
- What is meant by the period and frequency of an object executing uniform circular motion? (junior and senior high school)
- What is a hertz? (junior and senior high school)
- What is the difference between the concepts of speed and velocity? (junior and senior high school)
- How can we calculate the speed of an object executing uniform circular motion? (junior and senior high school)
- What are the forces acting on Ozobot while Ozobot is executing uniform circular motion on a flat, level surface? (senior high school)
- Which two of these forces cancel one another and why? (senior high school)
- What is centripetal acceleration? (senior high school)
- What force accounts for the centripetal acceleration experienced by Ozobot? (senior high school)
- What is the defining equation for the coefficient of static friction? (senior high school)

Explanation of OzoCodes Used in This Lesson

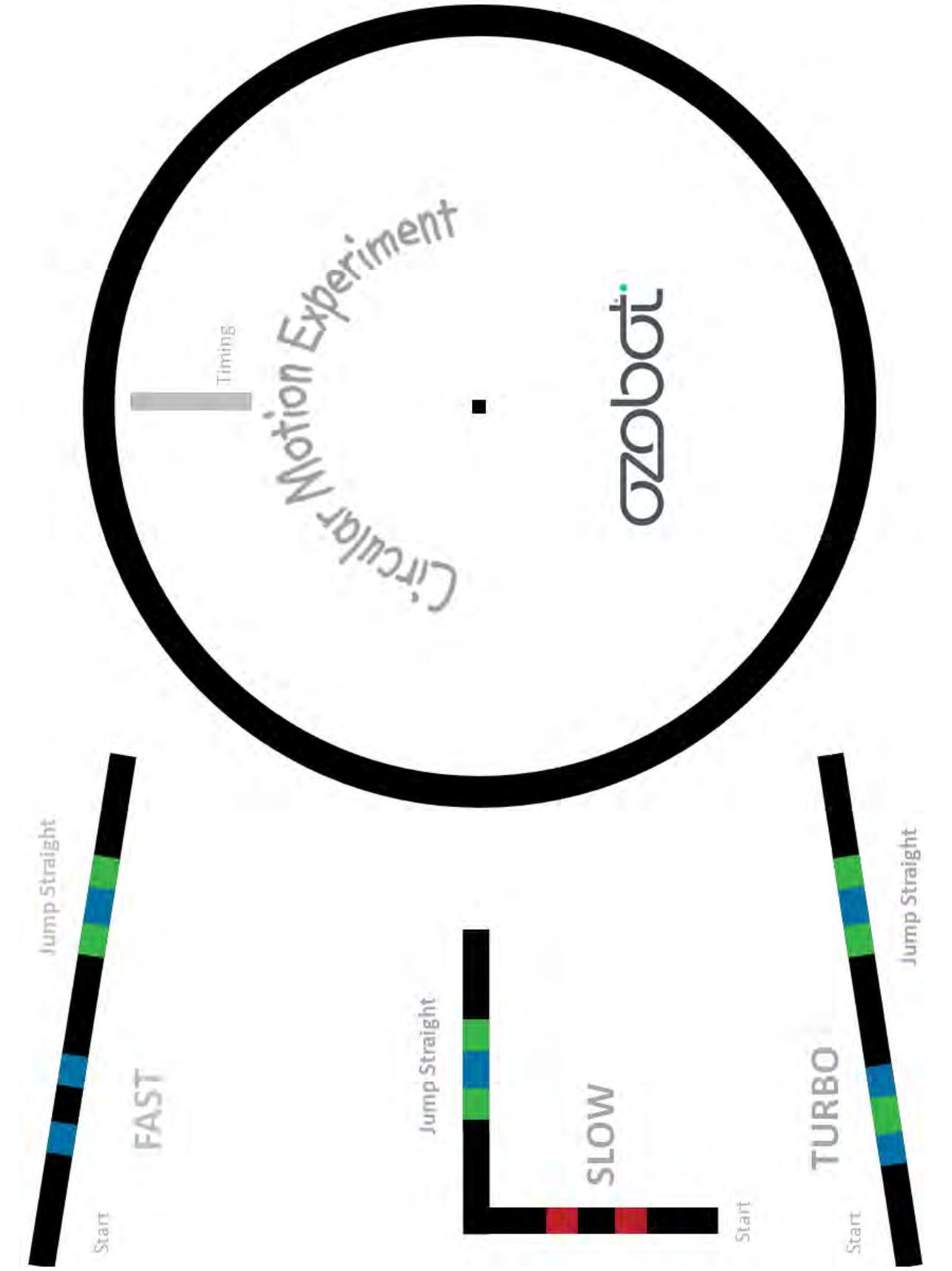
-  The *Jump Straight* symmetric OzoCode tells Ozobot to continue going straight when it reaches the end of a line.
-  *Slow* is one of the speed OzoCodes. It is a symmetric code that tells Ozobot to go somewhat slower than the default cruise speed.
-  *Fast* is one of the speed OzoCodes. It is a symmetric code that tells Ozobot to go somewhat faster than the default cruise speed.
-  *Turbo* is one of the speed OzoCodes. It is a symmetric code that tells Ozobot to go even faster than the "fast" speed.

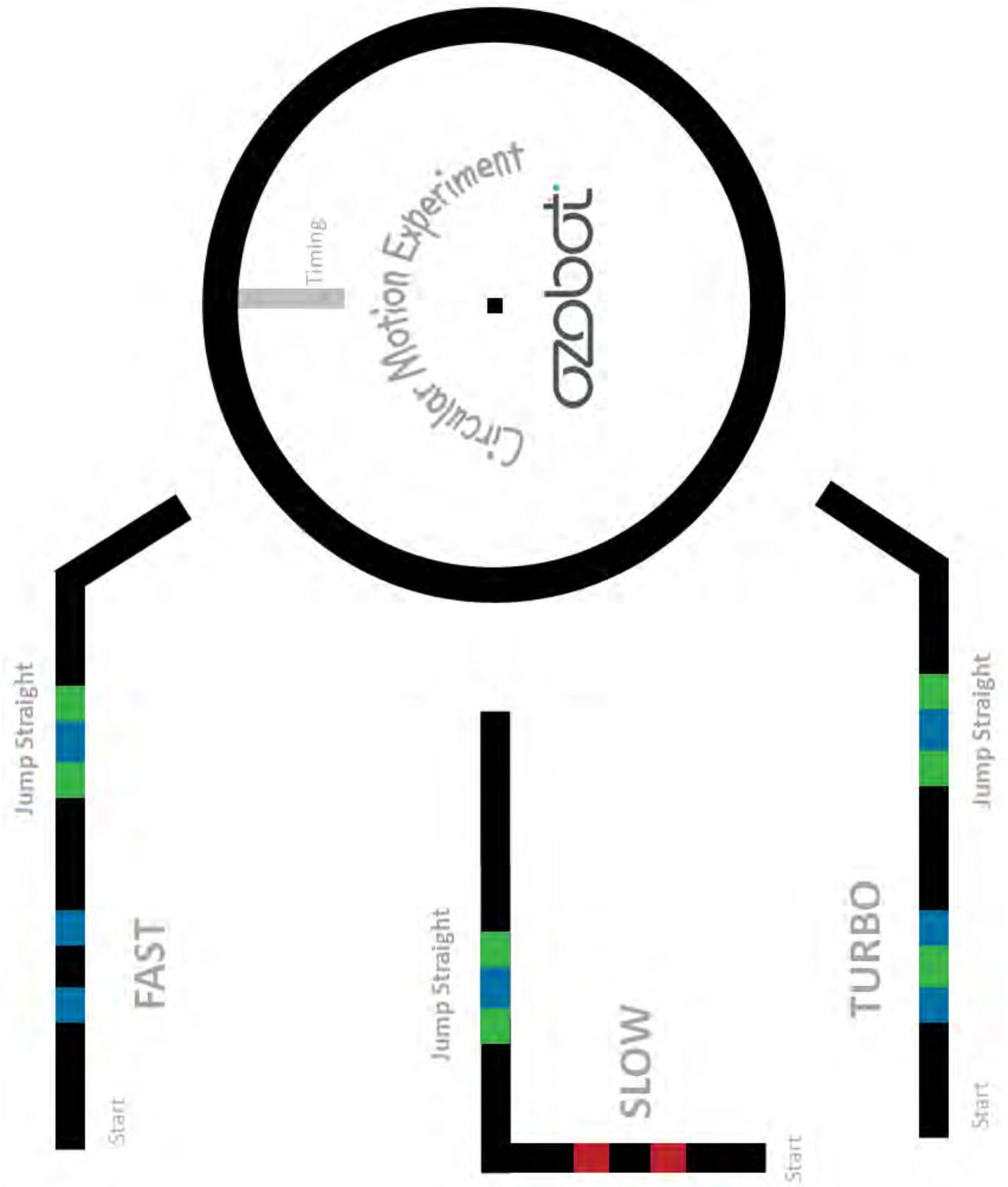
The OzoMaps for this Experiment

Pages 3 through 5 contain the OzoMaps that are needed for doing this experiment. The OzoMap of page 3 has Ozobot moving on a large circle, while the OzoMaps of pages 4 and 5 have Ozobot moving on medium and small circles, respectively. The ramps to the left of each of the circles make it easy to select the speed that Ozobot uses—either slow, fast, or turbo. To get Ozobot going at the desired speed, turn Ozobot off and then on again, and place Ozobot on the line at the position marked “Start”. The “Jump Straight” codes at the end of each ramp will then cause Ozobot to jump to the circle and begin traveling around the circle. To get Ozobot to travel at “Cruise” speed, turn Ozobot off and then on again, and place Ozobot directly on the circle.

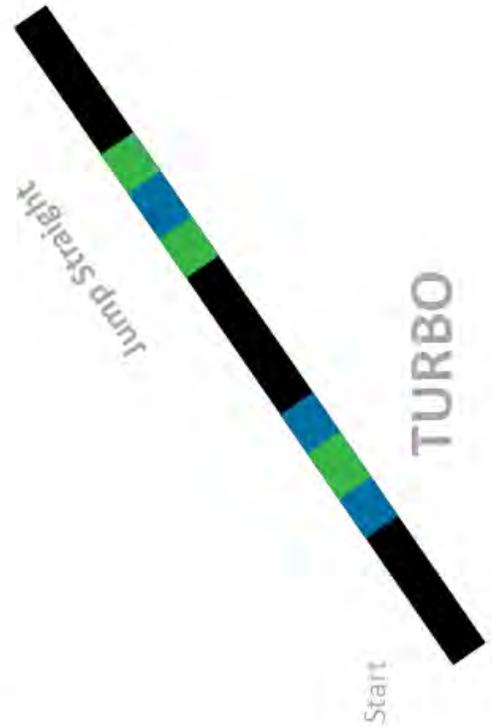
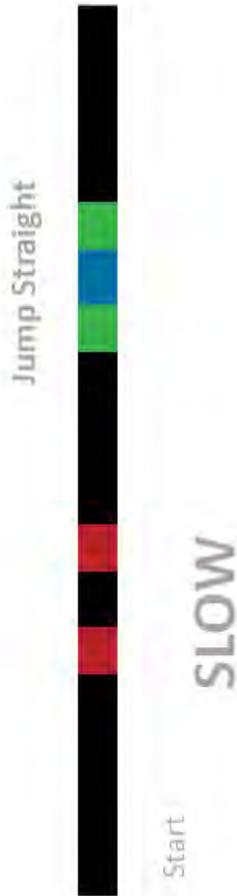
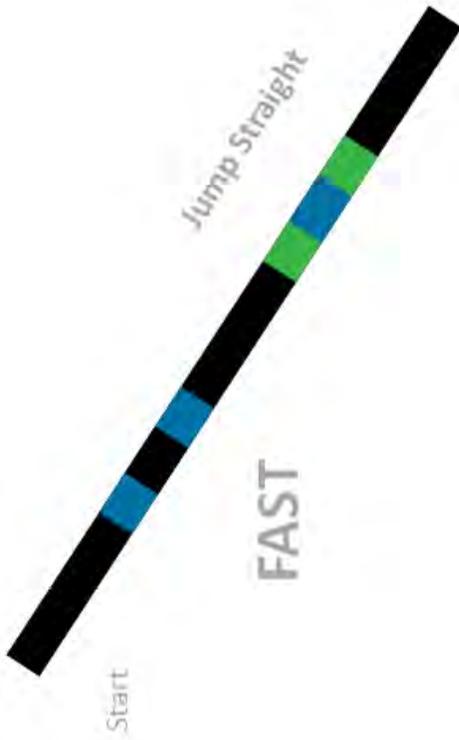
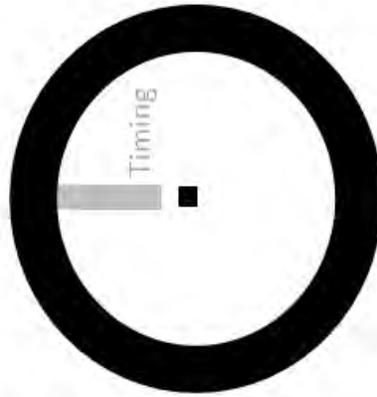
The tiny square at the center of each circle, as you might expect, marks the center of the circle. Rulers marked in centimeters, with millimeter divisions within the centimeters are ideal for measuring the radius of the circle. Since Ozobot is centered on the circular path, the radius is best measured from the center of the circle to the center of the solid circular path that Ozobot travels on.

A gray vertical line at the top of each circle is provided to help students determine Ozobot's *period* (time for one cycle) of revolution. It is best to time several, say 5 to 10 or more complete cycles, and then have the students divide the total time by the number of cycles in order to obtain the period of revolution. Once they have the period, they can determine the *frequency* by computing the *reciprocal* of the period. For example, if the period is 5 seconds, then the frequency is $1/5 = 0.2$ cycles/sec, or 0.2 hertz. Point out to the students that a *hertz*, abbreviated *Hz*, is the unit of frequency in the *International System of Units*, and that one hertz is defined as one cycle per second. Any stopwatch can be used for the time measurements. Also, most cell phones have built-in stopwatch capabilities. For example, the iPhone has a stopwatch feature in its clock app. The “seconds” hand of a wall clock can also be used for timing Ozobot, though perhaps with less accuracy than a stopwatch.



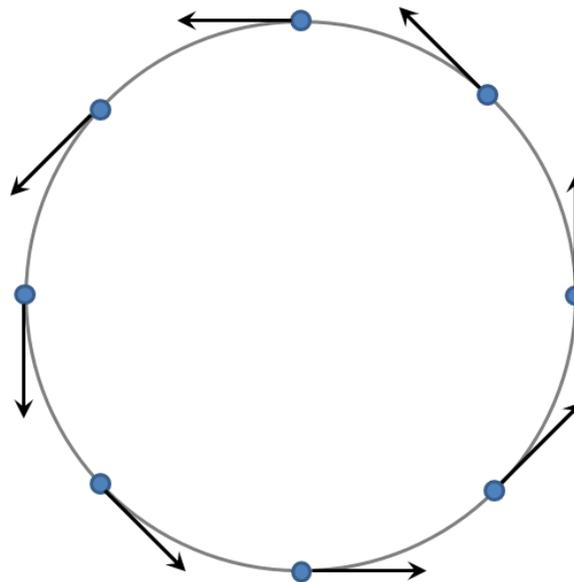


Circular Motion Experiment



Uniform Circular Motion

The concept of *uniform* circular motion means that the speed of the object moving in the circle is constant. There are many examples of uniform circular motion. A car moving on a curve at 50 miles per hour, a child on a merry-go-round at an amusement park, and the moon orbiting the earth are examples of nearly uniform circular motion. *Speed* tells us how fast an object is moving, while *velocity* tells us *not only* how fast, *but also* in what *direction* the object is moving. Velocity is therefore said to be *vector*, represented by an *arrowed line segment*. The velocity vector of an object moving in a circle depends upon where it is on the circle, but is always tangent to the circle, as shown in the figure below. Here, the length of the velocity vector is always the same, since the speed is constant as the object moves around the circle.



Data Collection

The Uniform Circular Motion Data Table on page 7 can be duplicated for students to record their data for this experiment. Depending upon the time that you wish to allot for this experiment, you can:

- Have each student group complete the entire table
- Assign student groups to one of Slow, Cruise, Fast, or Turbo speeds
- Assign student groups to one of the three circle size categories—small, medium, or large.

Uniform Circular Motion Data Table

Speed Category	Circle		Timing Data		Period (s)	Frequency (Hz)	Speed	
	Size Category	Radius (cm)	Circumference (cm)	Number of Cycles			Time (s)	(cm/s)
Slow	Small							
	Medium							
	Large							
Cruise	Small							
	Medium							
	Large							
Fast	Small							
	Medium							
	Large							
Turbo	Small							
	Medium							
	Large							

Discussion of How to Fill in the Data Table

An example of the data table completely filled in is shown on page 9. The speed categories are slow, cruise, fast, and turbo—the built-in speeds of Ozobot. The circle size categories are large, medium, and small, corresponding to the OzoMaps on pages 3 through 5, respectively. We will now discuss how the data for each of the columns is obtained, using the “slow, small” row as an example. You can carry out similar discussions with the students in your class.

- **Radius (cm)** A ruler marked in centimeters with millimeter divisions is used to measure the radius of the circle, from the center of the circle to the center of the thick line that Ozobot follows. The value of 1.95 cm indicates that the reading was to the nearest half of a millimeter. Note that the exact value obtained may be affected by the scale in which the OzoMaps are printed by your printer.
- **Circumference (cm)** Here, the students need to make use of geometry that they have learned. The circumference C of a circle is the distance around the circle and is given by the formula $C = 2\pi R$, where R is the radius. In this example, $2\pi R = 2 \times 3.14159 \times 1.95 \text{ cm} = 12.25 \text{ cm}$. Ozobot travels 12.25 cm each time it goes around the circle.
- **Number of Cycles** This is the number of times that Ozobot has gone around the circle while the student times Ozobot. In this example, Ozobot was timed for 5 cycles.
- **Time (s)** This is the time in seconds (s) required for Ozobot to make the indicated number of cycles and is typically measured by the use of a stopwatch. In this example, it took Ozobot 38.87 seconds to travel 5 cycles.
- **Period (s)** The period of a circulating object is the time required for one cycle. This is obtained by dividing the Time by the Number of Cycles. In this example, we have $38.87 \text{ s} / 5 = 7.77 \text{ s}$. Note that it is common to use an upper-case T to represent period in formulas. We could say that the period $T = 7.77 \text{ s}$.
- **Frequency (Hz)** Here, the student will need to make use of some math terminology that they may have studied. Frequency f is the reciprocal of the period T . Specifically, $f = 1 / T$. In this example, $f = 1 / 7.77 \text{ s} = 0.1287$ cycles per second. At this slow speed, Ozobot is making only slightly more than one-tenth of a cycle per second.

Uniform Circular Motion Data Table

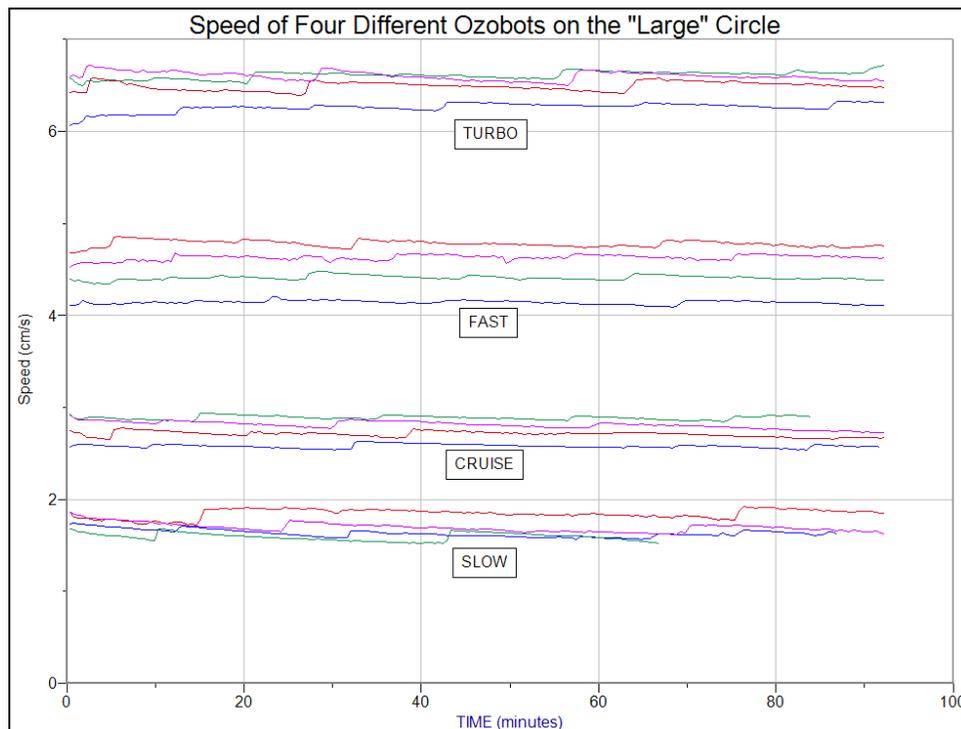
Speed Category	Circle		Timing Data		Period (s)	Frequency (Hz)	Speed		
	Size Category	Radius (cm)	Circumference (cm)	Number of Cycles			Time (s)	(cm/s)	(m/s)
Slow	Small	1.95	12.25	5	38.87	7.77	0.1287	1.58	0.0158
	Medium	4.20	26.39	5	75.85	15.17	0.0659	1.74	0.0174
	Large	6.50	40.84	5	116.87	23.37	0.0428	1.75	0.0175
Cruise	Small	1.95	12.25	10	52.90	5.29	0.1890	2.32	0.0232
	Medium	4.20	26.35	5	52.55	10.51	0.0951	2.51	0.0251
	Large	6.50	40.84	5	77.90	15.58	0.0642	2.62	0.0262
Fast	Small	1.95	12.25	20	68.84	3.44	0.2907	3.56	0.0356
	Medium	4.20	26.39	10	60.38	6.04	0.1656	4.37	0.0437
	Large	6.50	40.84	10	91.55	9.16	0.1092	4.46	0.0446
Turbo	Small	1.95	12.25	20	49.85	2.49	0.4014	4.92	0.0492
	Medium	4.20	26.39	10	48.05	4.81	0.2079	5.49	0.0549
	Large	6.50	40.84	10	65.32	6.53	0.1531	6.25	0.0625

- **Speed (cm/s)** Here the student needs to make use of the physical science concept of speed. *Speed s* is how fast an object is moving and is obtained by dividing the distance d traveled by the time t required to travel that distance. In a formula we could write this as $s = d / t$. With uniform circular motion, an object travels the circumference C in a time of period T . Therefore, $s = C / T = 12.25 \text{ cm} / 7.77 \text{ s} = 1.58 \text{ cm/s}$.
- **Speed (m/s)** Converting speed from cm/s to m/s, requires students to understand the relationship between centimeters (cm) and meters (m). There are 100 centimeters in a meter. Here is a good opportunity to introduce students to the concept of *dimensional analysis*, if they have not yet encountered it in their studies:

$$1.58 \frac{\cancel{\text{cm}}}{\text{s}} \times \frac{\text{m}}{100 \cancel{\text{cm}}} = 0.0158 \frac{\text{m}}{\text{s}}$$

In this dimensional analysis, the cm's cancel out, leaving only m/s.

It is important for the teacher to note that the speed of Ozobot will vary somewhat from bot-to-bot. These speed differences could be explained by small timing differences between Ozobots. The author of this teacher's guide has done a study of Ozobot speed from fully charged to automatic shut-down for four different Ozobots. The results are shown in the chart below.



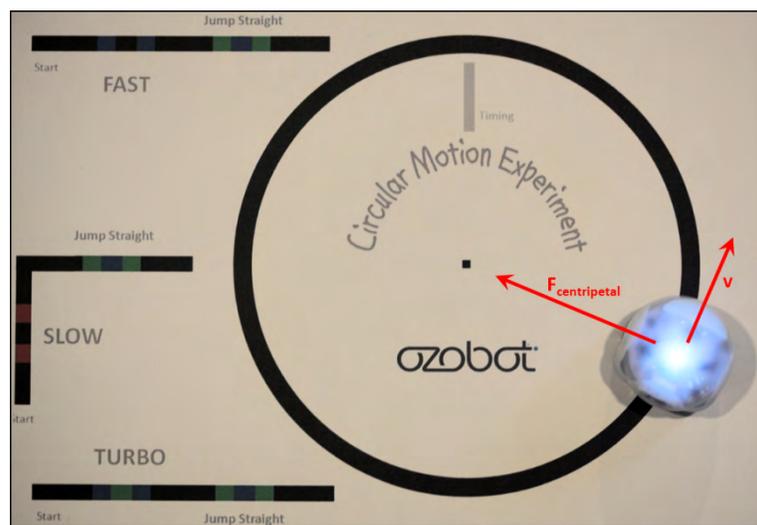
The chart gives you a feel for the possible range of speeds that students might obtain for each of the four built-in Ozobot speeds. The chart also shows that there is a slight variance of speed as time progresses for a given Ozobot. These speed differences over time are small enough that they are likely to be unnoticed visually or within the accuracy of student stopwatches.

Finally, you will notice upon studying the filled-in data table on page 9, that Ozobot's speed appears to decrease somewhat as the size of the circle changes from large, to medium, and to small. You might want to have the students hypothesize possible reasons for this.

What Causes Uniform Circular Motion

If you tie a washer to a string, you can swing it around over your head. What keeps it going in a circle? You can feel the tension in the string. The string is providing an inward force toward the center of the circular orbit of the washer. Meanwhile the velocity vector is always tangent to the circular orbit, and is perpendicular to the force vector. The inward force is known as the *centripetal* force, where "centripetal" comes from Latin words meaning "center seeking". Whenever an object follows a curved path, there is a centripetal force acting on the object.

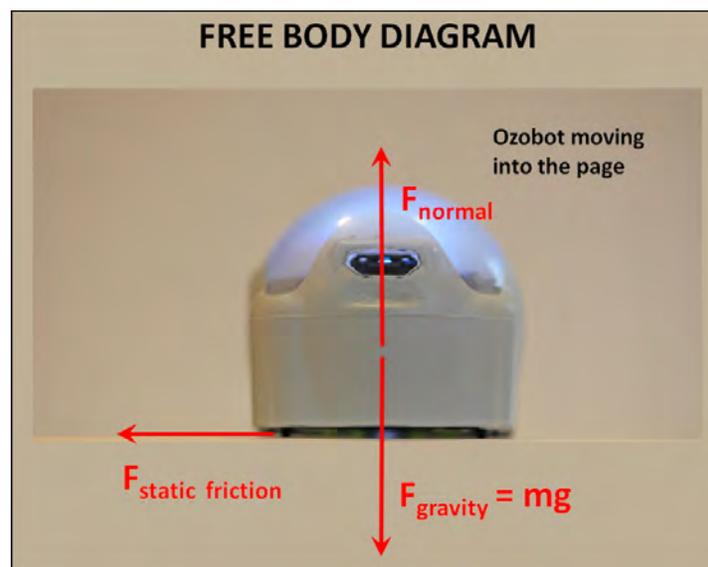
So what causes the centripetal force that keeps Ozobot in its circular path? There is no string attached to Ozobot. The friction between Ozobot's wheels and the surface on which Ozobot is traveling, either paper or the glass surface of a tablet, is directed toward the center of the circle, providing the centripetal force. See the picture below for clarification.



The Physics of Uniform Circular Motion

The remainder of this lesson is intended for high school students who are studying physics.

The figure below shows a free body diagram of the forces acting on Ozobot while in uniform circular motion. Since Ozobot remains on its level surface and does not move upward, the normal force and the force of gravity are equal in magnitude and oppositely directed. The net force is therefore the force of static friction between the wheels and surface on which Ozobot is traveling.



Physics tells us that the magnitude of the centripetal acceleration is given by v^2/r , where v is the magnitude of the velocity (i.e., the speed) and r is the radius of the circle. Newton's Second Law of Motion tells us that $F_{\text{net}} = ma$, where F_{net} is the net force, m is the mass of the object, and a is its acceleration. With Ozobot traveling on a level circle, the net force is the centripetal force and we have

$$F_{\text{centripetal}} = F_{\text{static friction}} = mv^2 / r$$

Physics students should be able to compute the centripetal force (equivalently, the force of static friction) if they know the speed, the radius of the circle, and the mass of an Ozobot. If a balance is available, students can measure the mass of an Ozobot directly. If not, students can assume that the mass of a typical Ozobot is about 20 grams (g), or 0.020 kilogram (kg) in the International System of Units. The extended data table on page 13 can be duplicated for student use.

Extended Uniform Circular Motion Data Table

Speed Category	Circle		Speed (m/sec)	Centripetal Acceleration (m/s ²)	Mass (kg)	Centripetal Force (N)
	Size Category	Radius (m)				
Slow	Small					
	Medium					
	Large					
Cruise	Small					
	Medium					
	Large					
Fast	Small					
	Medium					
	Large					
Turbo	Small					
	Medium					
	Large					

Discussion of How to Fill in the Extended Data Table

An example of the data table completely filled in is shown on page 15. The speed categories and circle size categories are the same as in the original data table. We will now discuss how the data for each of the columns is obtained, using the “slow, small” row as an example, as was done earlier. Depending on the depth of knowledge of your physics students, you can carry out similar discussions with the students in your class, if necessary.

- **Radius (m)** The values for Radius (cm) from the original data table are simply divided by 100 to convert the radius to meters (m). We want all quantities to be in the *International System of Units*. For example, 1.95 cm converts to 0.0195 m.
- **Speed (m/s)** These values are simply copied directly from the Speed (m/s) column from the original data table.
- **Centripetal Acceleration (m/s²)** Since centripetal acceleration is determined from v^2/r , the square of Speed (m/s) is divided by the Radius (m). For example, $(0.0158 \text{ m/s})^2 / 0.0195 \text{ m} = 0.01281 \text{ m/s}^2$.
- **Mass (kg)** The Ozobot used in collecting the example data had a mass of 19.9 g, which converts to 0.0199 kg, since a kg is 1/1000 of a g.
- **Centripetal Force (N)** Since $F_{\text{centripetal}} = mv^2 / r$, we multiply Mass (kg) by Centripetal Acceleration (m/s²) to obtain the centripetal force. For example, $0.0199 \text{ kg} \times 0.01281 \text{ m/s}^2 = 0.000252 \text{ N}$ (newtons—the *International System of Units* for force. Since the force is quite small, we re-write it in *scientific notation* as $25.2 \times 10^{-5} \text{ N}$. This will allow for quicker visual comparison of the centripetal forces in the table.

Discussion Questions

Answers for the teacher are in *italic* and square brackets [] after each question.

- Pick one of the three circle sizes—small, medium, or large. By observation of the extended data table, what happens to the centripetal force as the speed increases? Explain this behavior based upon the equation for centripetal force. [*The centripetal force increases as the speed increases. The equation tells us that the centripetal force increases as the square of the speed.*]

Extended Uniform Circular Motion Data Table

Speed Category	Circle		Speed (m/sec)	Centripetal Acceleration (m/s ²)	Mass (kg)	Centripetal Force (N)
	Size Category	Radius (m)				
Slow	Small	0.0195	0.0158	0.01281	0.0199	25.2×10^{-5}
	Medium	0.0420	0.0174	0.00721	0.0199	14.3×10^{-5}
	Large	0.0650	0.0175	0.00471	0.0199	9.4×10^{-5}
Cruise	Small	0.0195	0.0232	0.02760	0.0199	54.9×10^{-5}
	Medium	0.0420	0.0251	0.01500	0.0199	29.9×10^{-5}
	Large	0.0650	0.0262	0.01560	0.0199	21.0×10^{-5}
Fast	Small	0.0195	0.0356	0.06499	0.0199	129.3×10^{-5}
	Medium	0.0420	0.0437	0.04547	0.0199	90.5×10^{-5}
	Large	0.0650	0.0446	0.03060	0.0199	60.9×10^{-5}
Turbo	Small	0.0195	0.0492	0.12414	0.0199	247.0×10^{-5}
	Medium	0.0420	0.0549	0.07176	0.0199	142.8×10^{-5}
	Large	0.0650	0.0625	0.06010	0.0199	119.6×10^{-5}

- **Pick one of the four speed categories—slow, cruise, fast, or turbo. By observation of the extended data table, what happens to the centripetal force as the radius of the circle increases? Explain this behavior based upon the equation for centripetal force. *[The centripetal force decreases as the radius of the circle increases. The equation tells us that the centripetal force is inversely proportion to the radius of the circle. Therefore, as the circle's radius increases, the centripetal force decreases.]***
- **What does the equation for centripetal force tell us about the relationship between the centripetal force and the mass of Ozobot. *[The equation tells us that the centripetal force is directly proportional to the mass of Ozobot. The greater the mass, the greater the centripetal force.]***

Coefficient of Static Friction

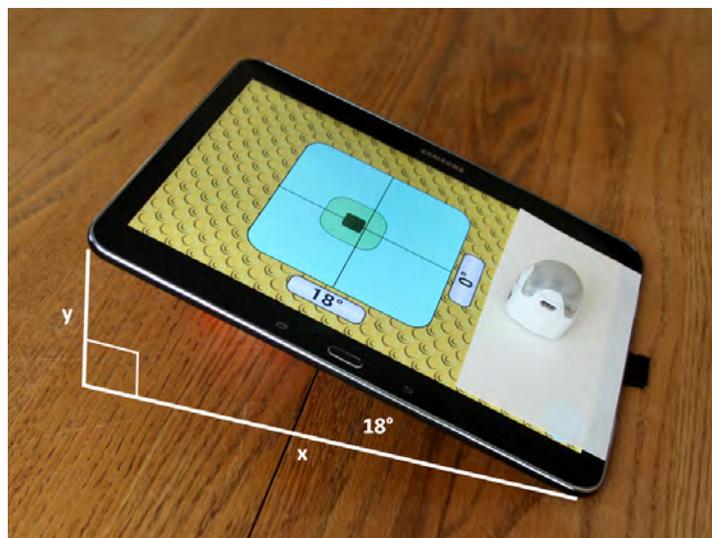
We have learned that the frictional force between Ozobot's wheels and the surface on which Ozobot travels is responsible for the centripetal force that keeps it going in a circle. More specifically, it is the force of *static friction* that keeps Ozobot from slipping to a larger radius circle. The *maximum* value that this force of static friction, ${}^{\max}F_{sf}$, can have is related to the normal force, F_n , by the equation:

$${}^{\max}F_{sf} = \mu_{sf} F_n .$$

μ_{sf} is known as the *coefficient of static friction*. It is empirically measured, and is defined for two surfaces that are at rest relative to one another. The normal force is a force exerted by each surface on the other surface and is perpendicular to the surface.

Measuring the Coefficient of Static Friction for Ozobot

There is a very easy way to determine the coefficient of static friction between Ozobot and the paper surface on which it travels for this circular motion experiment. As shown in the picture at the top of the next page, Ozobot is resting on a small piece of paper that has been taped to a tablet, with Ozobot's charge port facing you. Since the coefficient of static friction depends on the nature of the two contacting surfaces, the paper should be of the same type used to create the OzoMaps for this experiment. The tablet is running a free app called "*Tilt Meter*". The left side of the tablet is slowly and carefully lifted until Ozobot begins to slip down, noting the angle that the tablet is above the horizontal just before slipping begins.



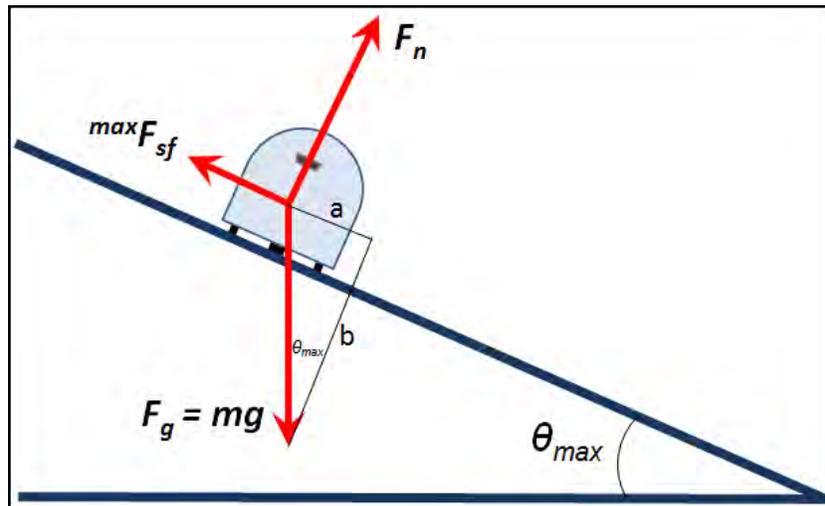
Note that you can use an iPhone or other Android phone rather than a tablet. The author of this lesson found that the angle θ_{max} at which Ozobot began slipping was about 32° for four different Ozobots. (The above picture shows Ozobot at an angle of 18° , well before Ozobot began slipping.) Remember that the angle may be different than this for your students, as not all paper surfaces are the same. Once you have this angle θ_{max} , the coefficient of static friction is simply $\tan \theta_{max}$, which is derived in the next section. With $\theta_{max} = 32^\circ$, $\tan 32^\circ \approx 0.62$. The coefficient of static friction μ_{sf} between Ozobot and the paper surface is about 0.62. Note that the coefficient of static friction is a dimensionless constant, which we would expect since by its definition, it is the ratio of two forces.

If you don't have either a tablet or cell phone with a Tilt Meter app, you can tape the paper to a piece of cardboard. In this case $\tan \theta_{max}$ is y/x , with x and y as shown in the above figure. (Students may need to be reminded of the trigonometric memory helper "soh cah toa", where tangent is the opposite over adjacent side of a right triangle.)

Derivation of the Formula $\mu_{sf} = \tan \theta_{max}$

The figure at the top of the next page shows a free body diagram for Ozobot sitting at rest on the tilted tablet just before slipping begins. There are two conditions:

1. *Ozobot does not move upward off of the incline surface.* This means that the component of the force of gravity perpendicular to the incline (shown as b in the diagram) must balance the normal force: $F_n = mg \cos \theta_{max}$.
2. *Ozobot must not slip on the incline.* This means that the component of the force of gravity parallel to the incline (shown as a in the diagram) must balance the force of static friction: $F_{sf} = mg \sin \theta_{max}$.



Therefore,

$$\max F_{sf} = \mu_{sf} F_n \rightarrow mg \sin \theta_{max} = \mu_{sf} mg \cos \theta_{max} \rightarrow \mu_{sf} = \sin \theta_{max} / \cos \theta_{max} = \tan \theta_{max}.$$

Maximum Safe Speed on an Unbanked Curve

So now let's get back to Ozobot traveling on the unbanked circular paper paths of this experiment. Knowing the coefficient of static friction, we can now determine the *maximum speed that Ozobot could go without slipping off of the path.* This speed is beyond the capabilities of Ozobot's current electronics, but will lead to some practical examples with real automobiles!

Referring to the free body diagram on page 12, since Ozobot does not accelerate upward, then the normal force must balance the force of gravity, i.e., $F_n = mg$. Since the force of static friction provides the centripetal force:

$$F_{centripetal} = \mu_{sf} F_n = \mu_{sf} mg = mv^2 / r \rightarrow \mu_{sf} g = v^2 / r \rightarrow v_{max} = \sqrt{\mu_{sf} g r}.$$

The maximum allowable safe speed before slipping would occur on an unbanked curve is proportional to the square root of the coefficient of static friction μ_{sf} , the value of acceleration due to gravity g , and the radius r of the circle.

Student Exercises (with Example solutions in square brackets [])

1. Using the coefficient of static friction that you obtained with Ozobot on the paper surface, determine the maximum speed that Ozobot could have on the small circle without slipping. $v_{max} = \sqrt{\mu_{sf} g r} = \sqrt{0.62 \left(9.8 \frac{m}{s^2}\right) (0.0195 m)} = 0.34 \text{ m/s}$. *This is about six times greater than Ozobot's turbo speed.]*
2. What would your answer be to the previous question if you did the experiment with Ozobot in a colony on the earth's moon? *[The acceleration due to gravity on the moon is 1/6 of that on the earth, or about 1.6 m/s². $v_{max} = \sqrt{\mu_{sf} g r} = \sqrt{0.62 \left(1.6 \frac{m}{s^2}\right) (0.0195 m)} = 0.019 \text{ m/s}$. Ozobot's slow speed is less than this, so Ozobot would be fine with slow speed on the small circle on the moon. Ozobot's cruise, fast, and turbo speeds are all greater than 0.019 m/s, so Ozobot would slip on a small circle on the moon.]*
3. Assume the following table of values for coefficients of static friction:

Surfaces	Coefficient of Static Friction
Rubber on dry concrete	0.75
Rubber on wet concrete	0.45
Rubber on water ice	0.05

Determine the maximum possible safe speed in miles/hour for a car traveling on an unbanked curve whose radius is 50 meters for each of these surface conditions. *[Answers: Approximately 43 mi/hr, 33 mi/hr, and 12 mi/hr, respectively. As one would expect, the dry concrete allows the greatest maximum speed, while the icy road allows the smallest safe maximum speed. Note: 1 m/s is equivalent to 2.237 mi/hr.]*

STEM Topics

- Interdisciplinary—physics, math and robotics working together
- Computer Science—colored visual codes are use to program a line-following robot
- Apply domain-specific vocabulary when communicating science, technology, engineering and mathematics concepts
- Engage in inquiry and critical thinking to answer complex questions

Materials

Ozobots (1 per every 3 to 4 students), fully charged)

8½" x 11" bright white cardstock for printing OzoMaps on pages 3, 4, and 5 for student lab groups

Paper copies of the data sheets on page 7 for each student lab group

Paper copies of the data sheet on page 13 for each student lab group (senior high school only)

Balance (optional) for measuring the mass of an Ozobot (senior high school only)

Free Tilt Meter app (optional—for senior high school only)

Estimated time-frame

Approximately 45 minutes (junior high)

Approximately 75 minutes (senior high school)

Vocabulary

Physical Science (Physics)	Mathematics	Measurement
acceleration	circumference	balance
centripetal acceleration	cos	centimeter (cm)
centripetal force	directly proportional	gram (g)
coefficient of static friction	division	hertz (Hz)
cycle	inversely proportional	kilogram (kg)
dimensional analysis	multiplication	meter (m)
dimensionless constant	perpendicular	millimeter (mm)
distance	pi (π)	newton (N)
force of gravity	radius	stopwatch
free body diagram	reciprocal	tilt meter
frequency	sin	
International System of Units	soh cah toa	
magnitude	square root	
mass	tan	
net force	tangent to	
normal force	vector	
period		
scientific notation		
Second Law of Motion		
speed		
static frictional force		
uniform circular motion		
velocity		