SUMMER MATH REVIEW PACKET

Dear Prospective Calculus Honor Students and Guardians,

We're excited to welcome you to Calculus Honor!

Attached is an **optional Summer Math Review Packet** designed for students whose registration has been approved. While completing this packet is not required, we strongly recommend it as a valuable opportunity to prepare for the course.

The goal of this review is to help you strengthen key concepts from Algebra and Precalculus—skills that are essential for success in Calculus. The packet includes practice problems and helpful online resources to guide your review and support you if you get stuck. Answers are included at the end so you can check your work and make corrections as needed.

Students who complete the packet and show all work on a separate sheet of paper will be eligible for extra credit if submitted during the second week of school.

In our experience, students who take time to review over the summer are better prepared, more confident, and tend to excel in the course. On the other hand, those who skip this step sometimes encounter difficulties later.

We suggest starting the packet sometime in July, once you've had a chance to enjoy a well-earned break. This will help ease your transition into the school year, reduce stress, and lighten your nightly homework load.

We hope you have a fun, restful summer, and we look forward to exploring the exciting world of Calculus with you this fall!

Warm regards,
The Calculus Teaching Team

Name ______________________________________

Your Task this summer:

1. REVIEW important ideas from PRECALCULUS and ALGEBRA.
2. REVIEW and MEMORIZE (again!) the UNIT CIRCLE.
3. PREPARE SOLUTIONS to the problems in this packet and bring your questions to the first week of class.

SEE RESOURCES BELOW TO HELP YOU!

HYPERDOC for Mastery and Review
Precalculus Review

Objective	Instructional Resources
Precalculus Review	• Functions and Properties • Point-Slope Form of Linear Equation • Secant Line Slope (average rate of change) • Interval Notation • Factoring Basics • Factoring Quadratics • Trigonometry Review - Unit Circle • Trigonometry Review - Inverse Trig Functions • Graphing Calculator Skills for Calculus (TI Family) • Math is Fun visual dictionary

Directions: See the HYPERDOC on page 2 for resources on MASTERY & REVIEW of these topics, definitions, and notation. Complete solutions to problems 1 – 3, and then review material in problem 4. Answers - not solutions - are provided at the end, so you may check your work. Make sure you understand the solution behind each answer. Assume that the problems do NOT require a graphing calculator, unless otherwise stated.

1. Let's open with a quick Algebra "correct the mistake" activity.

True or false. If false, change what is underlined to make the statement true.

a. $\left(x^3\right)^4 = x^{\underline{12}}$ T F

b. $x^{\frac{1}{2}}x^3 = x^{\underline{\frac{3}{2}}}$ T F

c. $(x+3)^2 = \underline{x^2+9}$ T F

d. $\dfrac{x^2-1}{x-1} = \underline{x}$ T F

e. $(4x+12)^2 = \underline{16}(x+3)^2$ T F

f. $\underline{3} + 2\sqrt{x-3} = 5\sqrt{x-3}$ T F

2. Write the equation of a line using the given information. You may use either slope intercept form, $y = mx + b$, or point-slope form $y = m(x - x_1) + y_1$, of the equation of the line.

   a) through the point $(1, 3)$ with slope of -4.

   b) through the points $(1, 3)$ and $(-4, 2)$.

   c) through the point $(1, 3)$ parallel to the line $y = 6(x-2)+5$

   d) through the point $(1, 3)$ perpendicular to the line $y = 6(x-2)+5$

3. Given the **functions** $f(x) = x^2 - 4x \ and \ g(x) = 5 - x$, find each of the following:

a) $f(g(-1))$

b) $g(f(-1))$

c) $\frac{f(x)-f(3)}{x-3}$

d) $g^{-1}(-1)$

4. **Factor** the following algebraic expressions completely.

a) $x^2 + 3x + 2$

b) $x^2 + 3x - 4$

c) $x^3 - 3x^2$

d) $x^2 - 9y^2$

e) $sin^2 x - cos^2 x$

f) $2x^2 + 7x + 3$

5. State the **domain** of the following functions.

a) $f(x) = \sqrt{(x+2)}$

b) $g(x) = \frac{x+2}{x^2-4}$

6. Find all solutions to the following equations without a calculator.

a) $2x - 3 = 5$

b) $\frac{1}{3}x = \frac{5}{12}$

c) $(x + 7)(x - 1) = 0$

d) $5 - (2x - 1) = 10$

e) $x^3 = x^2$

f) $sin(3x) = \frac{1}{2}$, on the interval $[0, 2\pi]$

g) $x^2 + 4x - 21 = 0$

h) $(2x + 3)^2 = 9$

i) $log_3 81 = x$

j) $log_4 x = -3$

7. ***Precalculus Vocabulary you need to know!*** Classifying mathematical objects and assigning them attributes using math vocabulary is an important skill in Calculus. Read each question carefully. Each question is "multiple choice" but you need to know a lot of "math vocabulary words" to answer. If you do not know the meaning of one of the terms please see the resource hyperdoc for a link to "Math is Fun" which has a terrific math dictionary that focuses on visual definitions.

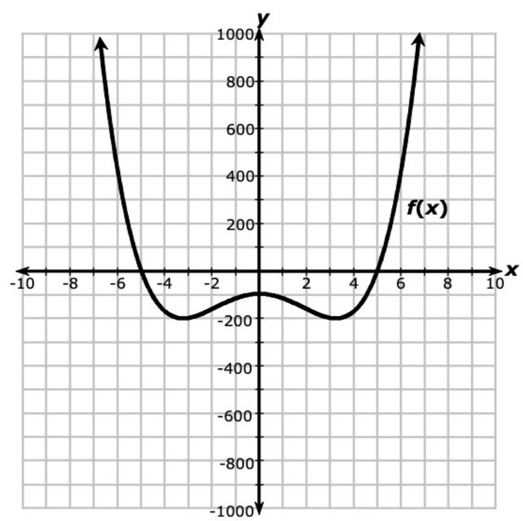
a. Understanding properties of **Quadratic Functions** is important in Calculus, as the quadratic function models velocity of a falling object.

Which quadratic function opens upwards and has a vertex at (0 , 3)?

(A) $y = -(x-3)^2$ (B) $y = (x-3)^2$ (C) $y = -x^2 + 3$ (D) $y = x^2 + 3$

b. **Polynomial functions** are seen often in Calculus. The statements below contain the term "extrema". Extrema is another way to say "turning points of a graph."

A polynomial function of degree four is graphed as shown.



Based on this graph, which statement is true?

A $f(x)$ has a total of four roots and three local extrema.

B $f(x)$ has a total of two roots and three local extrema.

C $f(x)$ has a total of two roots and five extrema.

D $f(x)$ has a total of four roots and five extrema.

c. This question uses "interval notation" in the options. See resource in HYPERDOC to learn about "interval notation".

What set of intervals describe the domain of the function $f(x) = \frac{x}{x^2+3x}$?

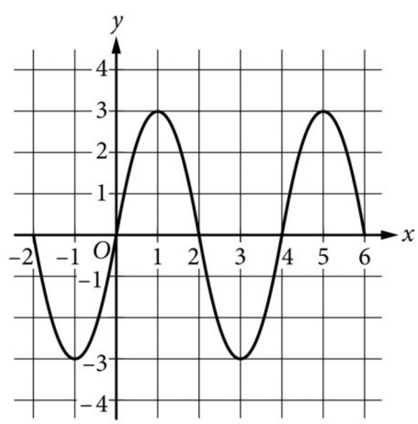
a. $(-\infty, -3)$ *and* $(-3, \infty)$

b. $(-\infty, -3]$ *and* $[-3, \infty)$

c. $(-\infty, -3]$, $[-3, 0)$ *and* $(0, \infty)$

d. $(-\infty, -3)$, $(-3, 0)$ *and* $(0, \infty)$

d. Interpreting graphs of functions is an important skill in Calculus. Choose the best response.



Graph of f

Let f be a sinusoidal function. The graph of $y = f(x)$ is given in the xy-plane. What is the period of f ?

(A) 2

(B) 3

(C) 4

(D) 6

e. Relating tables and math models is an important skill in Calculus. This question asks you to decide if a data set is modeled best as a linear or exponential function. Choose the best response.

t (months)	0	1	2	3	4
$P(t)$ (thousands)	20	30	45	67.5	101.25

The increasing function P gives the number of followers, in thousands, for a new musical group on a social media site. The table gives values of $P(t)$ for selected values of t, in months, since the musical group created their account on this social media site. If a model is constructed to represent these data, which of the following best applies to this situation?

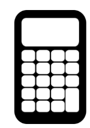
(A) $y = 10t + 20$

(B) $y = \frac{325}{16}t + 20$

(C) $y = 20\left(\frac{2}{3}\right)^t$

(D) $y = 20\left(\frac{3}{2}\right)^t$

8. ***Word Problems.*** Reading carefully and understanding which problem-solving strategy to use is an important skill in Calculus. CALCULATOR ACTIVE!!



a. Eli drops a ball from a height of 6 feet, and the return bounce is 82% of the previous height. How far has the ball traveled up and down when it hits the ground for the sixth time? Round the answer to the nearest tenth of a foot. (hint: A table could help you out) BONUS: can you write an equation that will predict how far the ball has traveled after x bounces?

b. For an art project, Alex takes a 22 inch by 28 inch poster board and cuts congruent squares out of the corners. She folds the poster board along the cuts to create a container with no top. All responses should include units.

i. If the size of the side length of the square cut is exactly 3 inches, what is the resulting height of the container?

ii. What is the resulting volume of the container?

iii. Write a function, V(x), which returns the volume of the container for a square cut out with side length, x. (hint: draw a picture)

9. ***Difference Quotients.*** Using function notation to write difference quotients is an important skill. If $f(x) = 3x^2 + 4x$, what is …?

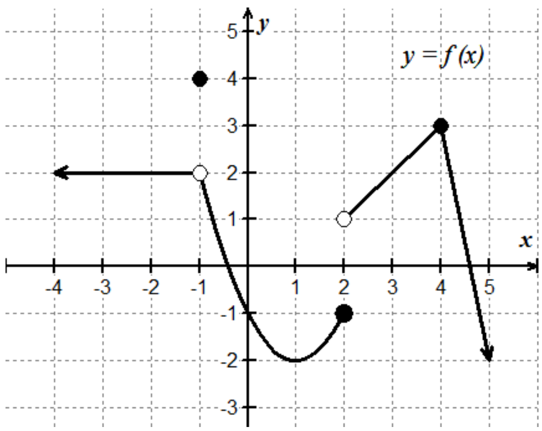
a. $\frac{f(4)-f(1)}{4-1}$

b. $\frac{f(x)-f(1)}{x-1}$

c. $f(x + h)$

d. $\frac{f(x+h)-f(x)}{h}$

10.



Use the graph above of $y = f(x)$ above to determine the following values.

A. $f(4)$

B. $f(-1)$

C. $f(2)$

11. ***Trigonometry Facts Review***

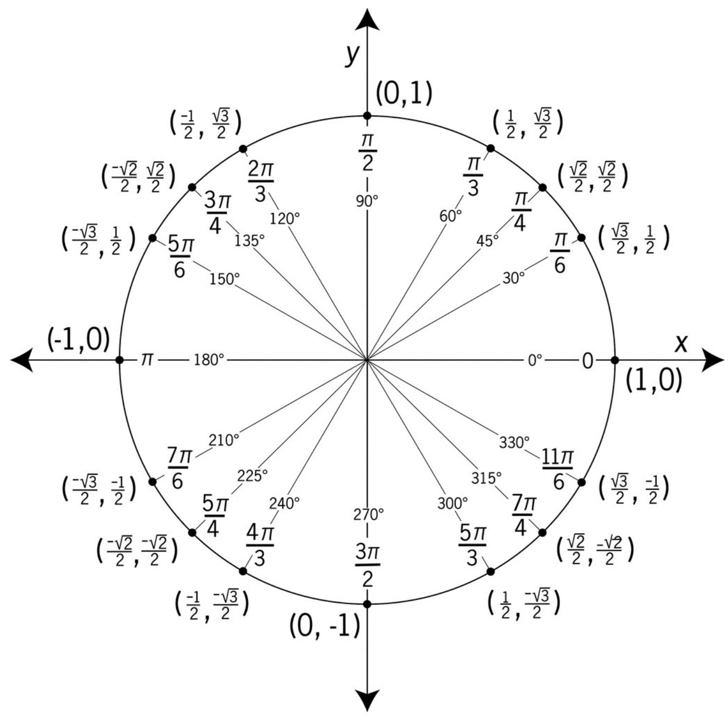
Take time this summer to review the values of sine, cosine, and tangent in the unit circle of the "cardinal angles" in radians. You may use whatever method helps you. All the information you are expected to know is in this unit circle.

<u>Having these committed to memory will help calculus run much smoother for you.</u>

Recall that each point on the unit circle

$(x, y) = (cos\ \theta,\ sin\ \theta)$

θ	$0°$ (or) 0	$30°$ (or) $\frac{\pi}{6}$	$45°$ (or) $\frac{\pi}{4}$	$60°$ (or) $\frac{\pi}{3}$	$90°$ (or) $\frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Not Defined



ANSWER KEY

1. a. True b. False c. False d. False e. True f. False

2. a. $y =- 4(x - 1) + 3$ (Point-Slope Form of Linear Equation)
   b. $y = 5(x - 1) + 3$
   c. $y = 6(x - 1) + 3$
   d. $y =- \frac{1}{6}(x - 1) + 3$

3. a. 12
   b. 0
   c. $x - 1$
   d. 6

4. a. $(x + 2)(x - 1)$
   b. $(x + 4)(x - 1)$
   c. $x(x^2 - 3)$
   d. $(x - 3y)(+ 3y)$
   e. $(\sin x - \cos x)(\sin x + \cos x)$
   f. $(x + 3)(2x + 1)$

5. a. $\{x \mid x \geq 2\}$ Roster notation OR $(2, \infty)$ Interval notation
   b. $\{x \mid x \neq - 2, 2\}$ Roster notation
   OR $(- \infty, - 2) \cup (- 2, 2) \cup (2, \infty)$ Interval notation

6. a. $x = 4$
   b. $x = \frac{5}{4}$
   c. $x =- 7 \; or \; x = 1$
   d. $x =- 2$
   e. $x = 0 \; or \; x = 1$
   f. $x = \frac{\pi}{18} \; or \; x = \frac{5\pi}{18}$
   g. $x =- 7 \; or \; x = 3$
   h. $x =- 3 \; or \; x = 0$
   i. $x = 4$
   j. $x = \frac{1}{64}$

7. a. D
   b. B
   c. D
   d. C
   e. D

8. a. 40.4 feet (HINT: total distance traveled is the sum of down and up bounce distances)
   b. (i) 3 inches (ii) 704 in^3 (iii) $V(x) = x(28 - 2x)(22 - 2x)$

9. a. 7 b. $3x + 7$ c. $3(x + h)^2 + 4(x + h) = 3x^2 + 6xh + 3h^2 + 4x + 4h$ d. $6x + 3h + 4$
10. a. 3 b. 4 c. -1