COMMON CORE

11-1 Graphs of Sine and Cosine

CC.9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph…* *Also* **CC.9-12.F.TF.5*, CC.9-12.F.BF.3, CC.9-12.F.IF.5*, CC.9-12.A.CED.2, CC.9-12.A.CED.3**

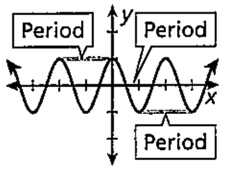
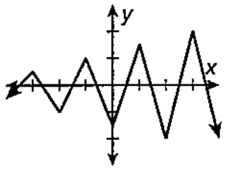
Objective
Recognize and graph periodic and trigonometric functions.

Vocabulary
periodic function
cycle
period
amplitude
frequency
phase shift

Why learn this?

Periodic phenomena such as sound waves can be modeled with trigonometric functions. (See Example 3.)

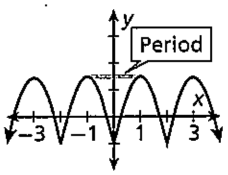
Periodic functions are functions that repeat exactly in regular intervals called **cycles**. The length of the cycle is called its **period**. Examine the graphs of the periodic function and nonperiodic function below. Notice that a cycle may begin at any point on the graph of a function.

Periodic	Not Periodic
	

EXAMPLE 1 Identifying Periodic Functions

Identify whether each function is periodic. If the function is periodic, give the period.

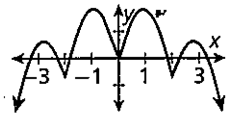
A



The pattern repeats exactly, so the function is periodic. Identify the period by using the start and finish of one cycle.

This function is periodic with period 2.

B

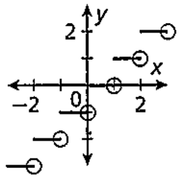


Although there is some symmetry, the pattern does not repeat exactly.

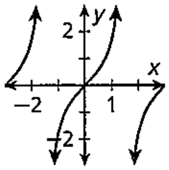
This function is not periodic.

CHECK IT OUT! **Identify whether each function is periodic. If the function is periodic, give the period.**

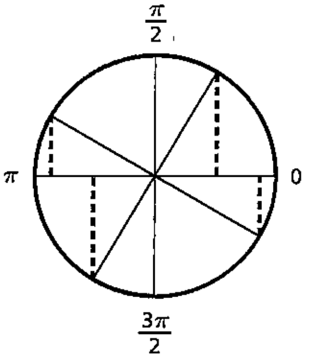
1a.



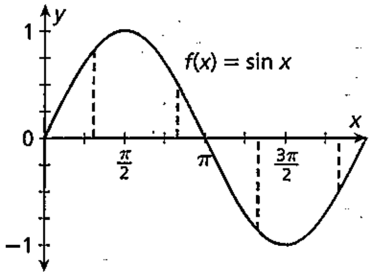
1b.



The trigonometric functions that you have studied are periodic. You can graph the function $f(x) = \sin x$ on the coordinate plane by using y-values from points on the unit circle where the independent variable x represents the angle θ in standard position.



$x(=\theta)$	y
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{5\pi}{6}$	$\frac{1}{2}$
$\frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$
$\frac{11\pi}{6}$	$-\frac{1}{2}$

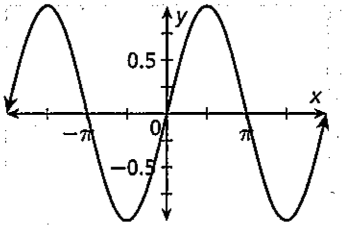
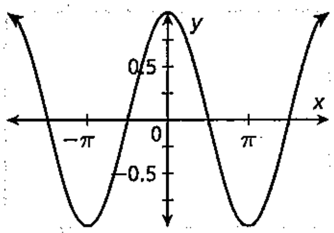


Similarly, the function $f(x) = \cos x$ can be graphed on the coordinate plane by using x-values from points on the unit circle.

The **amplitude** of sine and cosine functions is half of the difference between the maximum and minimum values of the function. The amplitude is always positive.

Know it! Note

Characteristics of the Graphs of Sine and Cosine

FUNCTION	$y = \sin x$	$y = \cos x$
GRAPH		
DOMAIN	$\{x \mid x \in \mathbb{R}\}$	$\{x \mid x \in \mathbb{R}\}$
RANGE	$\{y \mid -1 \le y \le 1\}$	$\{y \mid -1 \le y \le 1\}$
PERIOD	2π	2π
AMPLITUDE	1	1

Helpful Hint

The graph of the sine function passes through the origin. The graph of the cosine function has y-intercept 1.

You can use the parent functions to graph transformations $y = a \sin bx$ and $y = a \cos bx$. Recall that a indicates a vertical stretch $(|a| > 1)$ or compression $(0 < |a| < 1)$, which changes the amplitude. If a is less than 0, the graph is reflected across the x-axis. The value of b indicates a horizontal stretch or compression, which changes the period.

Know it! Note

Transformations of Sine and Cosine Graphs

For the graphs of $y = a \sin bx$ or $y = a \cos bx$ where $a \neq 0$ and x is in radians,

- the amplitude is $|a|$.
- the period is $\frac{2\pi}{|b|}$.

EXAMPLE 2 Stretching or Compressing Sine and Cosine Functions

Using $f(x) = \sin x$ as a guide, graph the function $g(x) = 3\sin 2x$. Identify th amplitude and period.

Step 1 Identify the amplitude and period.

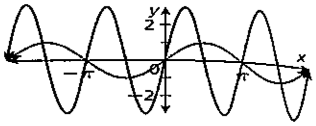
Because $a = 3$, the amplitude is $|a| = |3| = 3$.

Because $b = 2$, the period is $\frac{2\pi}{|b|} = \frac{2\pi}{|2|} = \pi$.

Step 2 Graph.

The curve is vertically stretched by a factor of 3 and horizontally compressed by a factor of $\frac{1}{2}$.

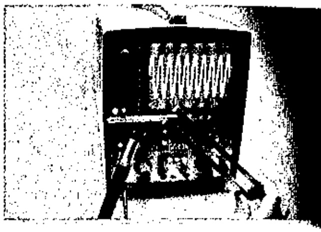
The parent function f has x-intercepts at multiples of π and g has x-intercepts at multiples of $\frac{\pi}{2}$.



The maximum value of g is 3, and the minimum value is −3.

CHECK IT OUT! **2.** Using $f(x) = \cos x$ as a guide, graph the function $h(x) = \frac{1}{3}\cos 2x$. Identify the amplitude and period.

Sine and cosine functions can be used to model real-world phenomena, such as sound waves. Different sounds create different waves. One way to distinguish sounds is to measure *frequency*. **Frequency** is the number of cycles in a given unit of time, so it is the reciprocal of the period of a function.

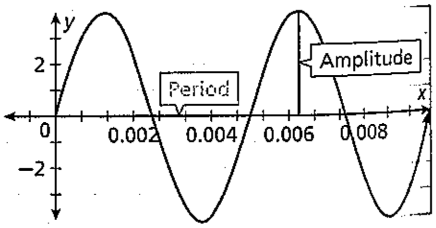


Hertz (Hz) is the standard measure of frequency and represents one cycle per second. For example, the sound wave made by a tuning fork for middle A has a frequency of 440 Hz. This means that the wave repeats 440 times in 1 second.

EXAMPLE 3 *Sound Application*

Use a sine function to graph a sound wave with a period of 0.005 second and an amplitude of 4 cm. Find the frequency in hertz for this sound wave.

Use a horizontal scale where one unit represents 0.001 second. The period tells you that it takes 0.005 seconds to complete one full cycle. The maximum and minimum values are given by the amplitude.



$$\text{frequency} = \frac{1}{\text{period}}$$
$$= \frac{1}{0.005} = 200 \text{ Hz}$$

The frequency of the sound wave is 200 Hz.

CHECK IT OUT! **3.** Use a sine function to graph a sound wave with a period of 0.004 second and an amplitude of 3 cm. Find the frequency in hertz for this sound wave.

Sine and cosine can also be translated as $y = \sin(x - h) + k$ and $y = \cos(x - h) + k$. Recall that a vertical translation by k units moves the graph up $(k > 0)$ or down $(k < 0)$.

A **phase shift** is a horizontal translation of a periodic function. A phase shift of h units moves the graph left $(h < 0)$ or right $(h > 0)$.

EXAMPLE 4 Identifying Phase Shifts for Sine and Cosine Functions

Using $f(x) = \sin x$ as a guide, graph $g(x) = \sin\left(x + \frac{\pi}{2}\right)$. Identify the x-intercepts and phase shift.

Step 1 Identify the amplitude and period.

Amplitude is $|a| = |1| = 1$.

The period is $\frac{2\pi}{|b|} = \frac{2\pi}{|1|} = 2\pi$.

Step 2 Identify the phase shift.

$x + \frac{\pi}{2} = x - \left(-\frac{\pi}{2}\right)$ *Identify h.*

Because $h = -\frac{\pi}{2}$, the phase shift is $\frac{\pi}{2}$ radians to the left.

All x-intercepts, maxima, and minima of $f(x)$ are shifted $\frac{\pi}{2}$ units to the left.

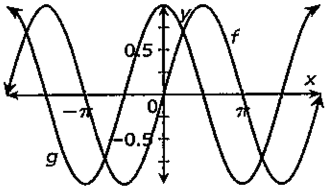
Step 3 Identify the x-intercepts.

The first x-intercept occurs at $-\frac{\pi}{2}$. Because $\sin x$ has two x-intercepts in each period of 2π, the x-intercepts occur at $-\frac{\pi}{2} + n\pi$, where n is an integer.

Step 4 Identify the maximum and minimum values.

The maximum and minimum values occur between the x-intercepts. The maxima occur at $2\pi n$ and have a value of 1. The minima occur at $\pi + 2\pi n$ and have a value of -1.

Step 5 Graph using all of the information about the function.



Helpful Hint

The repeating pattern is maximum, intercept, minimum, intercept,.... So intercepts occur twice as often as maximum or minimum values.

CHECK IT OUT! 4. Using $f(x) = \cos x$ as a guide, graph $g(x) = \cos(x - \pi)$. Identify the x-intercepts and phase shift.

You can combine the transformations of trigonometric functions. Use the values of a, b, h, and k to identify the important features of a sine or cosine function.

$$y = \overset{\text{Amplitude}}{\overset{\downarrow}{a}} \sin \underset{\underset{\text{Period}}{}}{\underset{\uparrow}{b}}\left(x - \overset{\text{Phase shift}}{\overset{\downarrow}{h}}\right) + \underset{\underset{\text{Vertical shift}}{}}{\underset{\uparrow}{k}}$$

EXAMPLE 5 *Entertainment Application*

The Ferris wheel at the landmark Navy Pier in Chicago takes 7 minutes to make one full rotation. The height H in feet above the ground of one of the six-person gondolas can be modeled by $H(t) = 70\sin\frac{2\pi}{7}(t - 1.75) + 80$, where t is time in minutes.

a. Graph the height of a cabin for two complete periods.

$$H(t) = 70\sin\frac{2\pi}{7}(t - 1.75) + 80 \quad a = 70,\ b = \frac{2\pi}{7},\ h = 1.75,\ k = 80$$

Step 1 Identify the important features of the graph.

Amplitude: 70

Period: $\frac{2\pi}{|b|} = \frac{2\pi}{\left|\frac{2\pi}{7}\right|} = 7$

The period is equal to the time required for one full rotation.

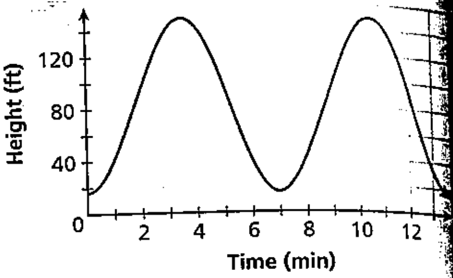
Phase shift: 1.75 minutes right

Vertical shift: 80

There are no x-intercepts.

Maxima: $80 + 70 = 150$ at 3.5 and 10.5

Minima: $80 - 70 = 10$ at 0, 7, and 14



Step 2 Graph using all of the information about the function.

b. What is the maximum height of a cabin?

The maximum height is $80 + 70 = 150$ feet above the ground.

CHECK IT OUT! 5. **What if...?** Suppose that the height H of a Ferris wheel can be modeled by $H(t) = -16\cos\frac{\pi}{45}t + 24$, where t is the time in seconds.

a. Graph the height of a cabin for two complete periods.

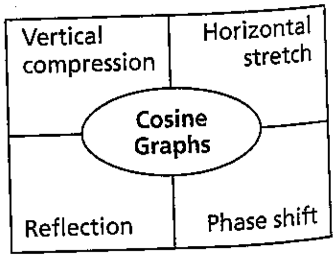
b. What is the maximum height of a cabin?

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. **DESCRIBE** how the frequency and period of a periodic function are related. How does this apply to the graph of $f(x) = \cos x$?
2. **EXPLAIN** how the maxima and minima are related to the amplitude and period of sine and cosine functions.
3. **GET ORGANIZED** Copy and complete the graphic organizer. For each type of transformation, give an example and state the period.

Know it! Note



11-1 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

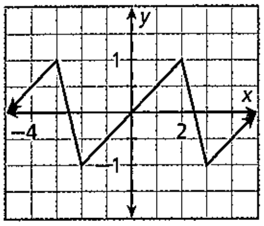
GUIDED PRACTICE

1. **Vocabulary** Periodic functions repeat in regular intervals called __?__. *(cycles* or *periods)*

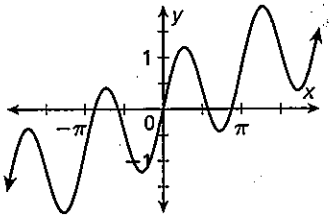
EXAMPLE 1

Identify whether each function is periodic. If the function is periodic, give the period.

2.



3.



EXAMPLE 2

Using $f(x) = \sin x$ or $f(x) = \cos x$ as a guide, graph each function. Identify the amplitude and the period.

4. $f(x) = 2\sin\frac{1}{2}x$ 5. $h(x) = \frac{1}{4}\cos x$ 6. $k(x) = \sin \pi x$

EXAMPLE 3

7. **Sound** Use a sine function to graph a sound wave with a period of 0.01 second and an amplitude of 6 in. Find the frequency in hertz for this sound wave.

EXAMPLE 4

Using $f(x) = \sin x$ or $f(x) = \cos x$ as a guide, graph each function. Identify the x-intercepts and the phase shift.

8. $f(x) = \sin\left(x + \frac{3\pi}{2}\right)$ 9. $g(x) = \cos\left(x - \frac{\pi}{2}\right)$ 10. $h(x) = \sin\left(x - \frac{\pi}{4}\right)$

EXAMPLE 5

11. **Recreation** The height H in feet above the ground of the seat of a playground swing can be modeled by $H(\theta) = -4\cos\theta + 6$, where θ is the angle that the swing makes with a vertical extended to the ground. Graph the height of a swing's seat for $0° \le \theta \le 90°$. How high is the swing when $\theta = 60°$?

PRACTICE AND PROBLEM SOLVING

pendent Practice

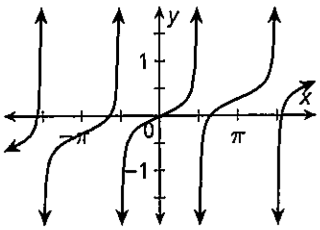
For ercises	See Example
12–13	1
14–17	2
18	3
9–22	4
23	5

tra Practice

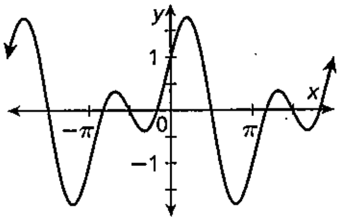
Extra Practice for
Skills Practice and
ications Practice
ses

Identify whether each function is periodic. If the function is periodic, give the period.

12.



13.



Using $f(x) = \sin x$ or $f(x) = \cos x$ as a guide, graph each function. Identify the amplitude and period.

14. $f(x) = 4\cos x$ 15. $g(x) = \frac{3}{2}\sin x$ 16. $g(x) = -\cos 4x$ 17. $j(x) = 6\sin\frac{1}{3}x$

18. **Sound** Use a sine function to graph a sound wave with a period of 0.025 seconds and an amplitude of 5 in. Find the frequency in hertz for this sound wave.

Using $f(x) = \sin x$ or $f(x) = \cos x$ as a guide, graph each function. Identify the x-intercepts and phase shift.

19. $f(x) = \sin(x + \pi)$

20. $h(x) = \cos(x - 3\pi)$

21. $g(x) = \sin\left(x + \frac{3\pi}{4}\right)$

22. $j(x) = \cos\left(x + \frac{\pi}{4}\right)$

23. **Oceanography** The depth d in feet of the water in a bay at any time is given by $d(t) = \frac{3}{2}\sin\left(\frac{5\pi}{31}t\right) + 23$, where t is the time in hours. Graph the depth of the water. What are the maximum and minimum depths of the water?

LINK

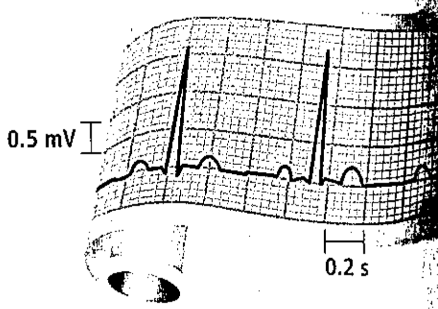
Medicine

An EKG measures the electrical signals that control the rhythm of a beating heart. EKGs are used to diagnose and monitor heart disease.

24. **Medicine** The figure shows a normal adult electrocardiogram, known as an EKG. Each cycle in the EKG represents one heartbeat.

- **a.** What is the period of one heartbeat?
- **b.** The pulse rate is the number of beats in one minute. What is the pulse rate indicated by the EKG?
- **c.** What is the frequency of the EKG?
- **d.** How does the pulse rate relate to the frequency in hertz?

Adult EKG



Determine the amplitude and period for each function. Then describe the transformation from its parent function.

25. $f(x) = \sin\left(x + \frac{\pi}{4}\right) - 1$

26. $h(x) = \frac{3}{4}\cos\frac{\pi}{4}x$

27. $h(x) = \cos(2\pi x) - 2$

28. $j(x) = -3\sin 3x$

Estimation Use a graph of sine or cosine to estimate each value.

29. $\sin 160°$

30. $\cos 50°$

31. $\sin 15°$

32. $\cos 95°$

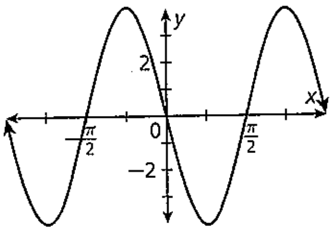
Write both a sine and a cosine function for each set of conditions.

33. amplitude of 6, period of π

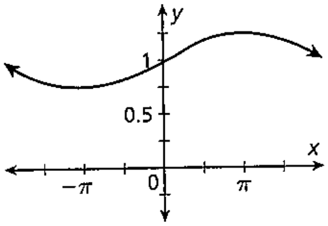
34. amplitude of $\frac{1}{4}$, phase shift of $\frac{2}{3}\pi$ left

Write both a sine and a cosine function that could be used to represent each graph.

35.



36.



MULTI-STEP TEST PREP

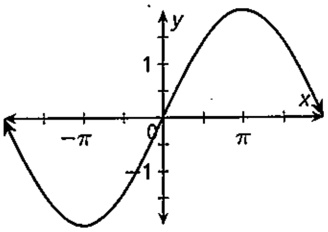
37. The tide in a bay has a maximum height of 3 m and a minimum height of 0 m. It takes 6.1 hours for the tide to go out and another 6.1 hours for it to come back in. The height of the tide h is modeled as a function of time t.

- **a.** What are the period and amplitude of h? What are the maximum and minimum values?
- **b.** Assume that high tide occurs at $t = 0$. What are $h(0)$ and $h(6.1)$?
- **c.** Write h in the form $h(t) = a\cos bt + k$.

38. **Critical Thinking** Given the amplitude and period of a sine function, can you find its maximum and minimum values and their corresponding x-values? If not, what information do you need and how would you use it?

39. **Write About It** What happens to the period of $f(x) = \sin b\theta$ when $b > 1$? $b < 1$? Explain.

TEST PREP



40. Which trigonometric function best matches the graph?

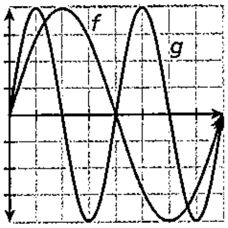
Ⓐ $y = \frac{1}{2}\sin x$ Ⓒ $y = \frac{1}{2}\sin 2x$

Ⓑ $y = 2\sin x$ Ⓓ $y = 2\sin\frac{1}{2}x$

41. What is the amplitude for $y = -4\cos 3\pi x$?

Ⓕ -4 Ⓗ 4

Ⓖ 3 Ⓙ 3π



42. Based on the graphs, what is the relationship between f and g?

Ⓐ f has twice the amplitude of g.

Ⓑ f has twice the period of g.

Ⓒ f has twice the frequency of g.

Ⓓ f has twice the cycle of g.

43. **Short Response** Using $y = \sin x$ as a guide, graph $y = -4\sin 2(x - \pi)$ on the interval $[0, 2\pi]$ and describe the transformations.

CHALLENGE AND EXTEND

44. Graph $f(x) = \text{Sin}^{-1} x$ and $g(x) = \text{Cos}^{-1} x$. (*Hint:* Use what you learned about graphs of inverse functions in a previous lesson and inverse trigonometric functions in a previous lesson.)

Consider the functions $f(\theta) = \frac{1}{2}\sin\theta$ and $g(\theta) = 2\cos\theta$ for $0° \leq \theta \leq 360°$.

45. On the same set of coordinate axes, graph $f(\theta)$ and $g(\theta)$.

46. What are the approximate coordinates of the points of intersection of $f(\theta)$ and $g(\theta)$?

47. When is $f(\theta) > g(\theta)$?

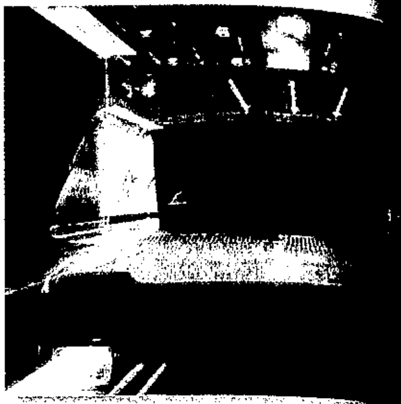
COMMON CORE

11-2 Graphs of Other Trigonometric Functions

CC.9-12.F.TF.5 Choose trigonometric functions to model periodic phenomena…* *Also* **CC.9-12.F.IF.5*, CC.9-12.F.IF.7*, CC.9-12.F.BF.3, CC.9-12.A.CED.2, CC.9-12.A.CED.3**

Objective
Recognize and graph trigonometric functions.

Why learn this?
You can use the graphs of reciprocal trigonometric functions to model rotating objects such as lights. (See Exercise 25.)

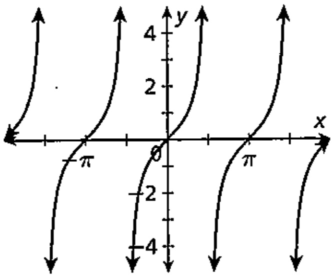
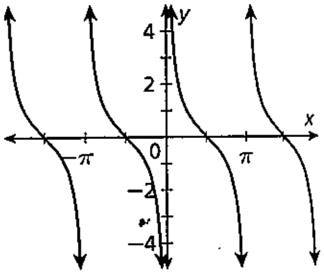


The tangent and cotangent functions can be graphed on the coordinate plane. The tangent function is undefined when $\theta = \frac{\pi}{2} + \pi n$, where n is an integer. The cotangent function is undefined when $\theta = \pi n$. These values are excluded from the domain and are represented by vertical asymptotes on the graph. Because tangent and cotangent have no maximum or minimum values, amplitude is undefined.

To graph tangent and cotangent, let the variable x represent the angle θ in standard position.

Know it! Note

Characteristics of the Graphs of Tangent and Cotangent

FUNCTION	$y = \tan x$	$y = \cot x$
GRAPH		
DOMAIN	$\left\{x \mid x \neq \frac{\pi}{2} + \pi n,\right.$ where n is an integer$\left.\right\}$	$\{x \mid x \neq \pi n,$ where n is an integer$\}$
RANGE	$\{y \mid -\infty < y < \infty\}$	$\{y \mid -\infty < y < \infty\}$
PERIOD	π	π
AMPLITUDE	undefined	undefined

Like sine and cosine, you can transform the tangent function.

Know it! Note

Transformations of Tangent Graphs

For the graph of $y = a \tan bx$, where $a \neq 0$ and x is in radians,

- the period is $\frac{\pi}{|b|}$.
- the asymptotes are located at $x = \frac{\pi}{2|b|} + \frac{\pi n}{|b|}$, where n is an integer.

EXAMPLE 1 Transforming Tangent Functions

Using $f(x) = \tan x$ as a guide, graph $g(x) = \tan 2x$. Identify the period, x-intercepts, and asymptotes.

Step 1 Identify the period.

Because $b = 2$, the period is $\frac{\pi}{|b|} = \frac{\pi}{|2|} = \frac{\pi}{2}$.

Step 2 Identify the x-intercepts.

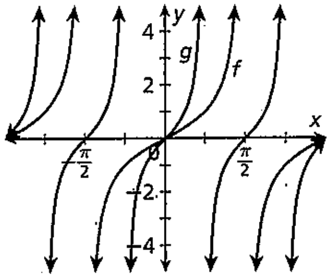
An x-intercept occurs at $x = 0$. Because the period is $\frac{\pi}{2}$, the x-intercepts occur at $\frac{\pi}{2}n$, where n is an integer.

Step 3 Identify the asymptotes.

Because $b = 2$, the asymptotes occur at $x = \frac{\pi}{2|2|} + \frac{\pi n}{|2|}$, or $x = \frac{\pi}{4} + \frac{\pi n}{2}$.

Step 4 Graph using all of the information about the function.

$g(x) = \tan 2x$



CHECK IT OUT! 1. Using $f(x) = \tan x$ as a guide, graph $g(x) = 3\tan\frac{1}{2}x$. Identify the period, x-intercepts, and asymptotes.

Know it! Note

Transformations of Cotangent Graphs

For the graph of $y = a\cot bx$, where $a \neq 0$ and x is in radians,

- the period is $\frac{\pi}{|b|}$.
- the asymptotes are located at $x = \frac{\pi n}{|b|}$, where n is an integer.

EXAMPLE 2 Graphing the Cotangent Function

Using $f(x) = \cot x$ as a guide, graph $g(x) = \cot 0.5x$. Identify the period, x-intercepts, and asymptotes.

Step 1 Identify the period.

Because $b = 0.5$, the period is $\frac{\pi}{|b|} = \frac{\pi}{|0.5|} = 2\pi$.

Step 2 Identify the x-intercepts.

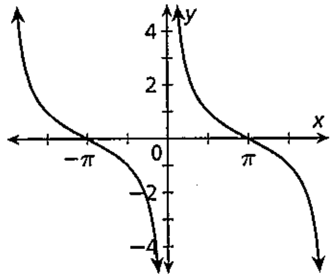
An x-intercept occurs at $x = \pi$. Because the period is 2π, the x-intercepts occur at $x = \pi + 2\pi n$, where n is an integer.

Step 3 Identify the asymptotes.

Because $b = 0.5$, the asymptotes occur at

$$x = \frac{\pi n}{|0.5|} = 2\pi n.$$

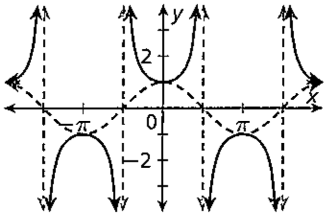
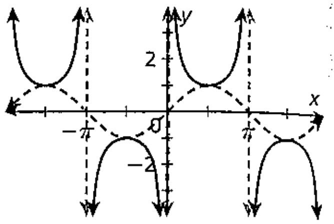
Step 4 Graph using all of the information about the function.



CHECK IT OUT! 2. Using $f(x) = \cot x$ as a guide, graph $g(x) = -\cot 2x$. Identify the period, x-intercepts, and asymptotes.

Recall that $\sec\theta = \frac{1}{\cos\theta}$. So, secant is undefined where cosine equals zero and the graph will have vertical asymptotes at those locations. Secant will also have the same period as cosine. Sine and cosecant have a similar relationship. Because secant and cosecant have no absolute maxima or minima, amplitude is undefined.

Characteristics of the Graphs of Secant and Cosecant

FUNCTION	$y = \sec x$	$y = \csc x$
GRAPH		
DOMAIN	$\left\{x \mid x \neq \frac{\pi}{2} + \pi n,\right.$ where n is an integer$\left.\right\}$	$\left\{x \mid x \neq \pi n,\right.$ where n is an integer$\left.\right\}$
RANGE	$\{y \mid y \leq -1, \text{ or } y \geq 1\}$	$\{y \mid y \leq -1, \text{ or } y \geq 1\}$
PERIOD	2π	2π
AMPLITUDE	undefined	undefined

You can graph transformations of secant and cosecant by using what you learned in the previous lesson about transformations of graphs of cosine and sine.

EXAMPLE 3 Graphing Secant and Cosecant Functions

Using $f(x) = \cos x$ as a guide, graph $g(x) = \sec 2x$. Identify the period and asymptotes.

Step 1 Identify the period.

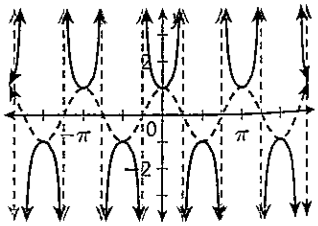
Because $\sec 2x$ is the reciprocal of $\cos 2x$, the graphs will have the same period.

Because $b = 2$ for $\cos 2x$, the period is $\frac{2\pi}{|b|} = \frac{2\pi}{|2|} = \pi$.

Step 2 Identify the asymptotes.

Because the period is π, the asymptotes occur at $x = \frac{\pi}{2|2|} + \frac{\pi}{|2|}n = \frac{\pi}{4} + \frac{\pi}{2}n$, where n is an integer.

Step 3 Graph using all of the information about the function.



CHECK IT OUT! 3. Using $f(x) = \sin x$ as a guide, graph $g(x) = 2\csc x$. Identify the period and asymptotes.

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. **EXPLAIN** why $f(x) = \sin x$ can be used to graph $g(x) = \csc x$.
2. **EXPLAIN** how the zeros of the cosine function relate to the vertical asymptotes of the graph of the tangent function.
3. **GET ORGANIZED** Copy and complete the graphic organizer.

Know it! Note

Function	Zeros	Asymptotes	Period
$y = \sec x$			
$y = \csc x$			
$y = \cot x$			
$y = \tan x$			

11-2 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

EXAMPLE 1

Using $f(x) = \tan x$ as a guide, graph each function. Identify the period, x-intercepts, and asymptotes.

1. $k(x) = 2\tan(3x)$ **2.** $g(x) = \tan\frac{1}{4}x$ **3.** $h(x) = \tan 2\pi x$

EXAMPLE 2

Using $f(x) = \cot x$ as a guide, graph each function. Identify the period, x-intercepts, and asymptotes.

4. $j(x) = 0.25\cot x$ **5.** $p(x) = \cot 2x$ **6.** $g(x) = \frac{3}{2}\cot x$

EXAMPLE 3

Using $f(x) = \cos x$ or $f(x) = \sin x$ as a guide, graph each function. Identify the period and asymptotes.

7. $g(x) = \frac{1}{2}\sec x$ **8.** $q(x) = \sec 4x$ **9.** $h(x) = 3\csc x$

PRACTICE AND PROBLEM SOLVING

ependent Practice

For ercises	See Example
10–13	1
14–16	2
17–19	3

tra Practice

Extra Practice for Skills Practice and plications Practice rcises.

Using $f(x) = \tan x$ as a guide, graph each function. Identify the period, x-intercepts, and asymptotes.

10. $p(x) = \tan\frac{3}{2}x$ **11.** $g(x) = \tan\left(x + \frac{\pi}{4}\right)$

12. $h(x) = \frac{1}{2}\tan 4x$ **13.** $j(x) = -2\tan\frac{\pi}{2}x$

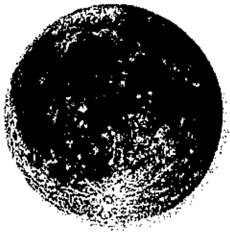
Using $f(x) = \cot x$ as a guide, graph each function. Identify the period, x-intercepts, and asymptotes.

14. $h(x) = 4\cot x$ **15.** $g(x) = \cot\frac{1}{4}x$ **16.** $j(x) = 0.1\cot x$

Using $f(x) = \cos x$ or $f(x) = \sin x$ as a guide, graph each function. Identify the period and asymptotes.

17. $g(x) = -\sec x$ **18.** $k(x) = \frac{1}{2}\csc x$ **19.** $h(x) = \csc(-x)$

MULTI-STEP TEST PREP



20. Between 1:00 P.M. ($t = 1$) and 6:00 P.M. ($t = 6$), the height (in meters) of the tide in bay is modeled by $h(t) = 0.4 \csc \frac{5\pi}{31} t$.

a. Graph the function for the range $1 \leq t \leq 6$.

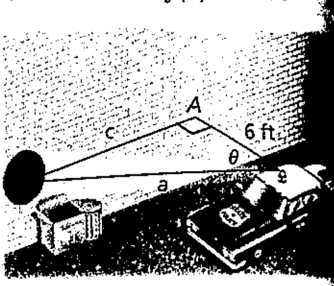
b. At what time does low tide occur?

c. What is the height of the tide at low tide?

d. What is the maximum height of the tide during this time span? When does this occur?

Find four values for which each function is undefined.

21. $f(\theta) = \tan \theta$ **22.** $g(\theta) = \cot \theta$ **23.** $h(\theta) = \sec \theta$ **24.** $j(\theta) = \csc \theta$



25. Law Enforcement A police car is parked on the side of the road next to a building. The flashing light on the car is 6 feet from the wall and completes one full rotation every 3 seconds. As the light rotates, it shines on the wall. The equation representing the distance a in feet is $a(t) = 6 \sec\left(\frac{2}{3}\pi t\right)$.

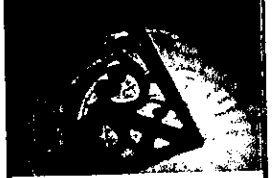
a. What is the period of $a(t)$?

b. Graph the function for $0 \leq t \leq 3$.

c. Critical Thinking Identify the location of any asymptotes. What do the asymptotes represent?

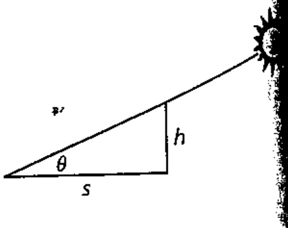
LINK

Math History



The Greek gnomon was a tall staff, but gnomon is also the part of a sundial that casts a shadow. Based on the variation of shadows at high noon, a gnomon can be used to determine the day of the year, in addition to the time of day.

26. Math History The ancient Greeks used a *gnomon*, a type of tall staff, to tell the time of day based on the lengths of shadows and the altitude θ of the sun above the horizon.



a. Use the figure to write a cotangent function that can be used to find the length of the shadow s in terms of the height of the gnomon h and the angle θ.

b. Graph your answer to part a for a gnomon of height 6 ft.

Complete the table by labeling each function as increasing or decreasing.

		$0 < x < \frac{\pi}{2}$	$\frac{\pi}{2} < x < \pi$	$\pi < x < \frac{3\pi}{2}$	$\frac{3\pi}{2} < x < 2\pi$
27.	sin*x*	☐	☐	☐	☐
28.	csc*x*	☐	☐	☐	☐
29.	cos*x*	☐	☐	☐	☐
30.	sec*x*	☐	☐	☐	☐
31.	tan*x*	☐	☐	☐	☐
32.	cot*x*	☐	☐	☐	☐

33. Critical Thinking Based on the table above, what do you observe about the increasing/decreasing relationship between reciprocal pairs of trigonometric functions?

34. **Critical Thinking** How do the signs (whether a function is positive or negative) of reciprocal pairs of trigonometric functions relate?

35. **Write About It** Describe how to graph $f(x) = 3\sec 4x$ by using the graph of $g(x) = 3\cos 4x$.

TEST PREP

36. Which is NOT in the domain of $y = \cot x$?

Ⓐ $-\frac{\pi}{2}$ Ⓑ 0 Ⓒ $\frac{\pi}{2}$ Ⓓ $\frac{3\pi}{2}$

37. What is the range of $f(x) = 3\csc 2\theta$?

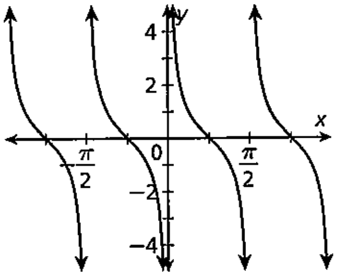
Ⓕ $\{y \mid y \le -1 \text{ or } y \ge 1\}$ Ⓗ $\{y \mid y \le -2 \text{ or } y \ge 2\}$

Ⓖ $\{y \mid y \le -3 \text{ or } y \ge 3\}$ Ⓙ $\{y \mid y \le -\frac{1}{2} \text{ or } y \ge \frac{1}{2}\}$

38. Which could be the equation of the graph?

Ⓐ $y = \tan 2x$ Ⓒ $y = 2\tan x$

Ⓑ $y = \cot 2x$ Ⓓ $y = 2\cot x$



39. What is the period of $y = \tan \frac{1}{2}x$?

Ⓕ $\frac{\pi}{2}$ Ⓗ 2π

Ⓖ π Ⓙ 4π

40. The graph of which function has a period of $\frac{2\pi}{3}$ and an asymptote at $x = \frac{\pi}{2}$?

Ⓐ $y = \sec \frac{3}{2}x$ Ⓒ $y = \csc \frac{3}{2}x$

Ⓑ $y = \sec 3x$ Ⓓ $y = \csc 3x$

CHALLENGE AND EXTEND

Describe the period, local maximum and minimum values, and phase shift.

41. $f(x) = 4 - 3\csc \pi(x - 1)$ 42. $g(x) = 4\cot \frac{1}{2}\left(x - \frac{\pi}{2}\right)$ 43. $h(x) = 0.5\sec 2\left(x + \frac{\pi}{4}\right)$

44. $f(x) = 9 + 2\tan 3(x + \pi)$ 45. $g(x) = 0.62 + 0.76\sec x$ 46. $h(x) = \csc \frac{\pi}{2}\left(x + \frac{5}{7}\right)$

Graph each trigonometric function and its inverse. Identify the domain and range of the corresponding inverse function.

47. $f(x) = \text{Sec}\, x$ for $0 \le x \le \pi$ and $x \ne \frac{\pi}{2}$ 48. $f(x) = \text{Tan}\, x$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

49. $g(x) = \text{Csc}\, x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ and $x \ne 0$ 50. $g(x) = \text{Cot}\, x$ for $0 < x < \pi$