# Parabolas

## Why learn this?

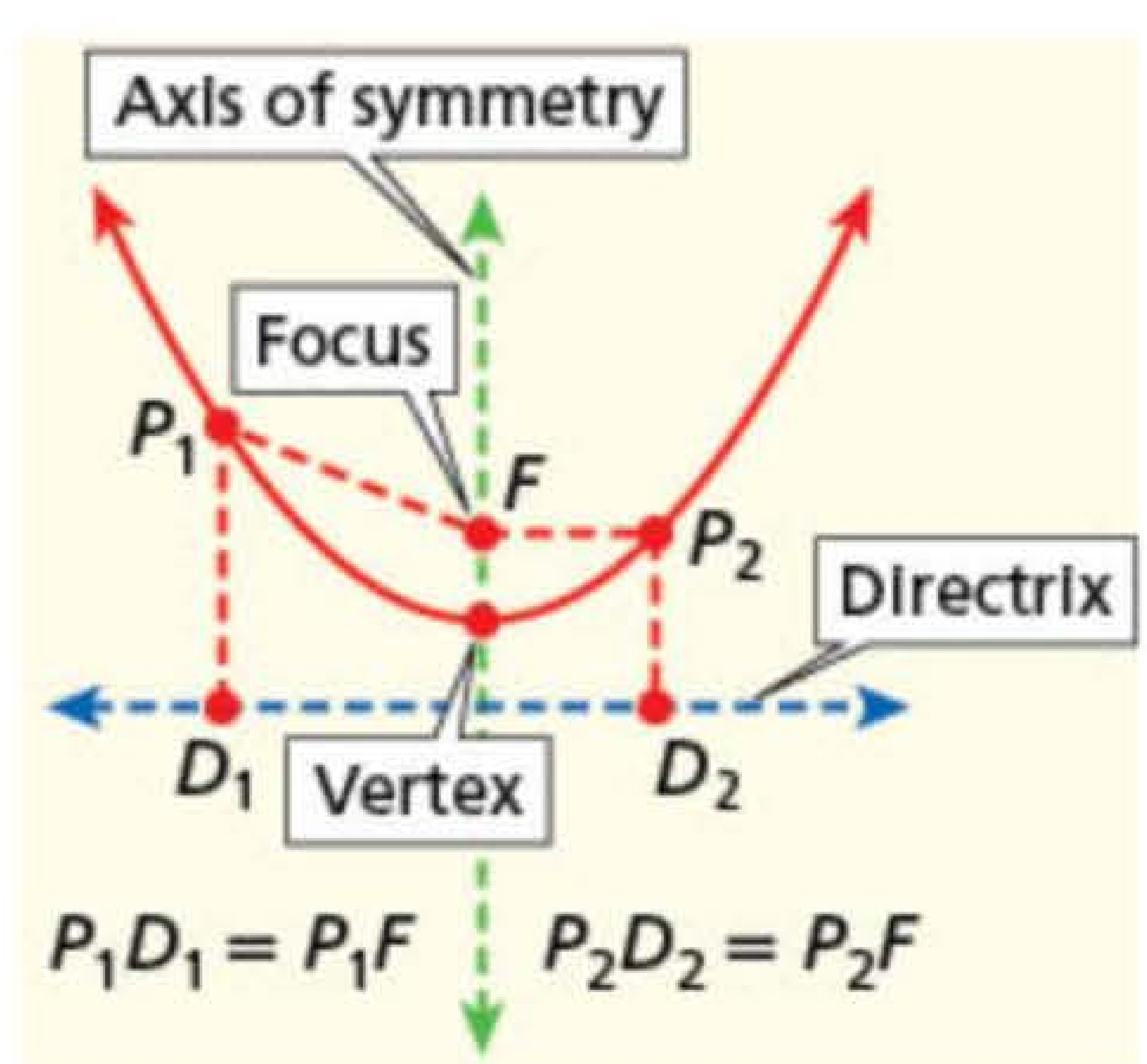
Parabolas are used with microphones to pick up sounds from sports events. (See Example 4.)

## Lesson Objective(s):

- Write the standard equation of a parabola and its axis of symmetry.
- Graph a parabola, and identify its focus, directrix, and axis of symmetry.

The graph of a quadratic function is a parabola. Because a parabola is a conic section, it can also be defined in terms of distance.

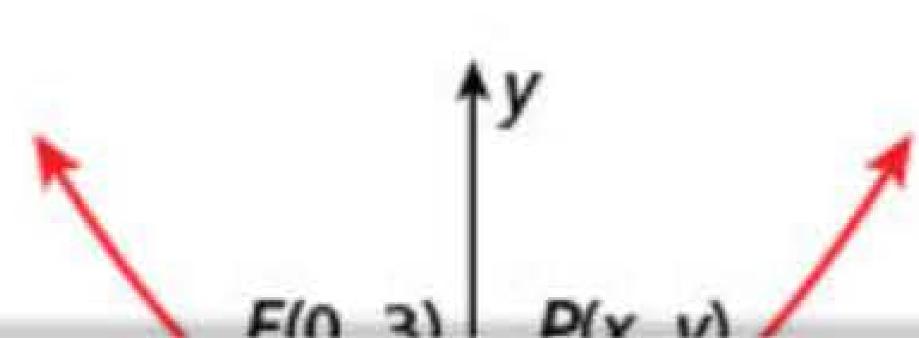
A parabola is the set of all points *P (x, y)* in a plane that are an equal distance from both a fixed point, the focus, and a fixed line, the directrix. A parabola has an axis of symmetry perpendicular to its directrix and that passes through its vertex. The vertex of a parabola is the midpoint of the segment connecting the focus and the directrix.




**EXAMPLE 1**

### Using the Distance Formula to Write the Equation of a Parabola

**Use the Distance Formula to find the equation of a parabola with focus $F(0, 3)$ and directrix $y = -3$.**

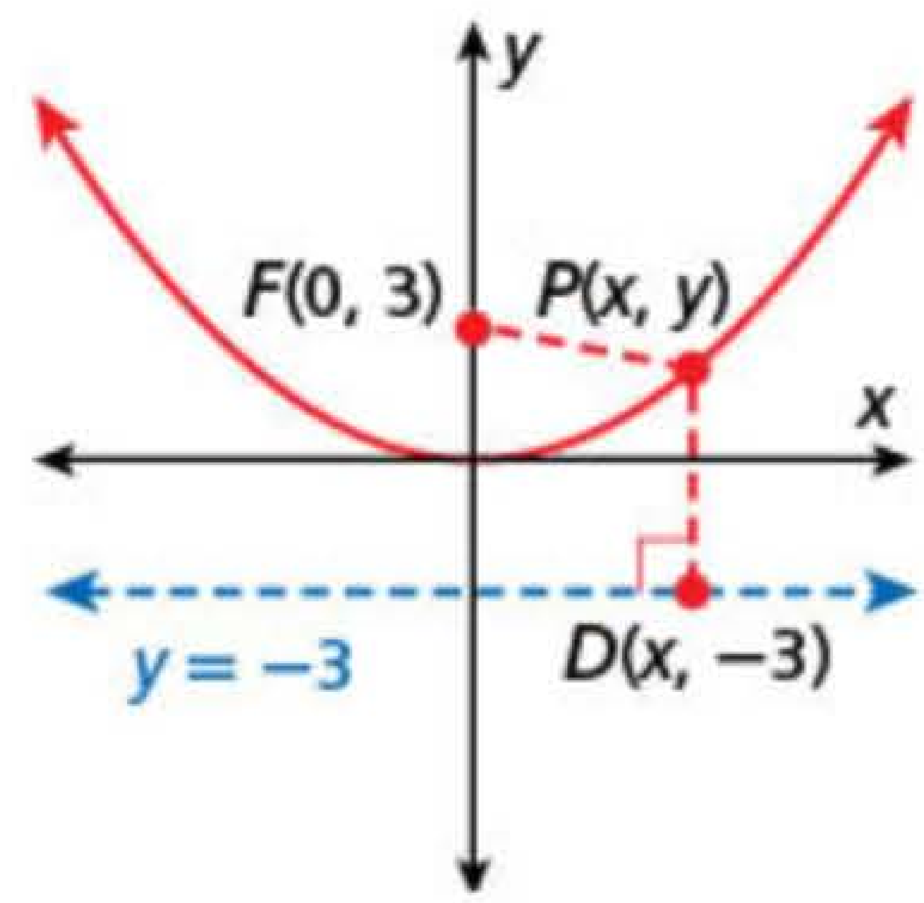


EXAMPLE 1

Remember

## Using the Distance Formula to Write the Equation of a Parabola

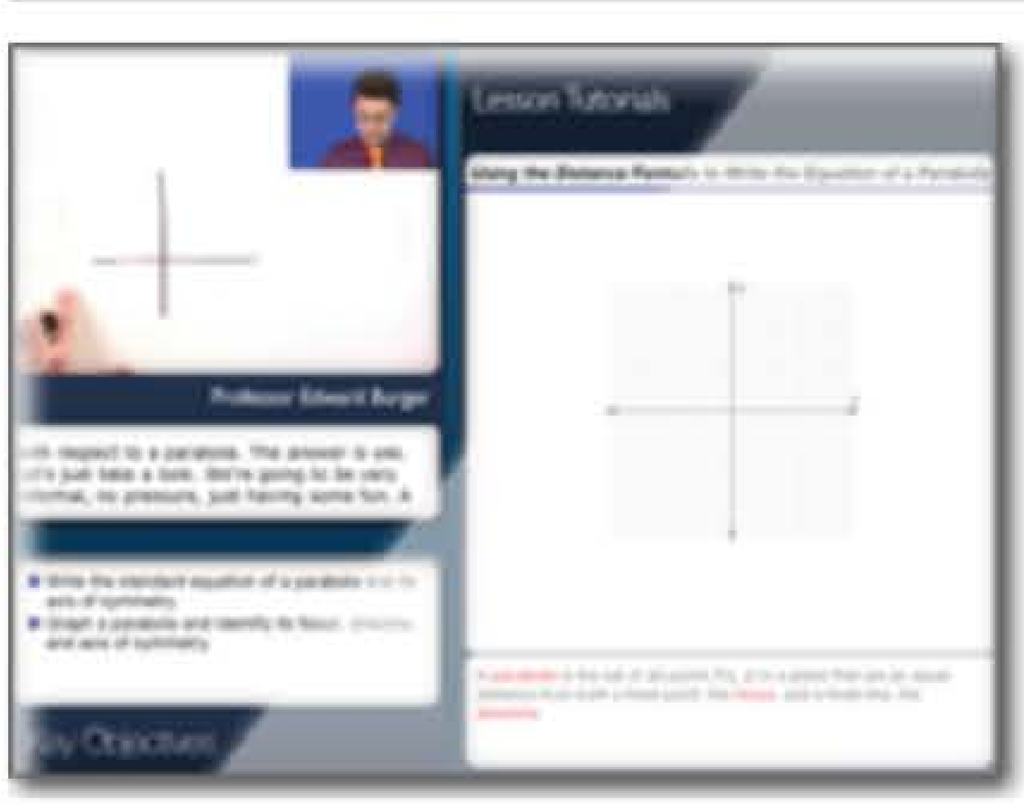
**Use the Distance Formula to find the equation of a parabola with focus $F(0, 3)$ and directrix $y = -3$.**




| | |
|---|---|
| $PF = PD$ | *Definition of a parabola* |
| $\sqrt{(x - x_1)^2 + (y - y_1)^2} = \sqrt{(x - x_2)^2 + (y - y_2)^2}$ | *Distance Formula* |
| $\sqrt{(x - 0)^2 + (y - 3)^2} = \sqrt{(x - x)^2 + (y + 3)^2}$ | *Substitute (0, 3) for $(x_1, y_1)$ and (x, −3) for $(x_2, y_2)$.* |
| $\sqrt{x^2 + (y - 3)^2} = \sqrt{(y + 3)^2}$ | *Simplify.* |
| $x^2 + (y - 3)^2 = (y + 3)^2$ | *Square both sides.* |
| $x^2 + y^2 - 6y + 9 = y^2 + 6y + 9$ | *Expand.* |
| $x^2 - 6y = 6y$ | *Subtract $y^2$ and 9 from both sides.* |
| $x^2 = 12y$ | *Add 6y to both sides.* |
| $y = \frac{1}{12}x^2$ | *Solve for y.* |

Tap the button to view Example 1 in StepReveal.

View in Motion



Tap the button to watch the lesson tutorial video for extra instruction.

Play Video

## Check It Out!

## Check It Out!

1. Use the Distance Formula to find the equation of a parabola with focus $F(0, 4)$ and directrix $y = -4$.

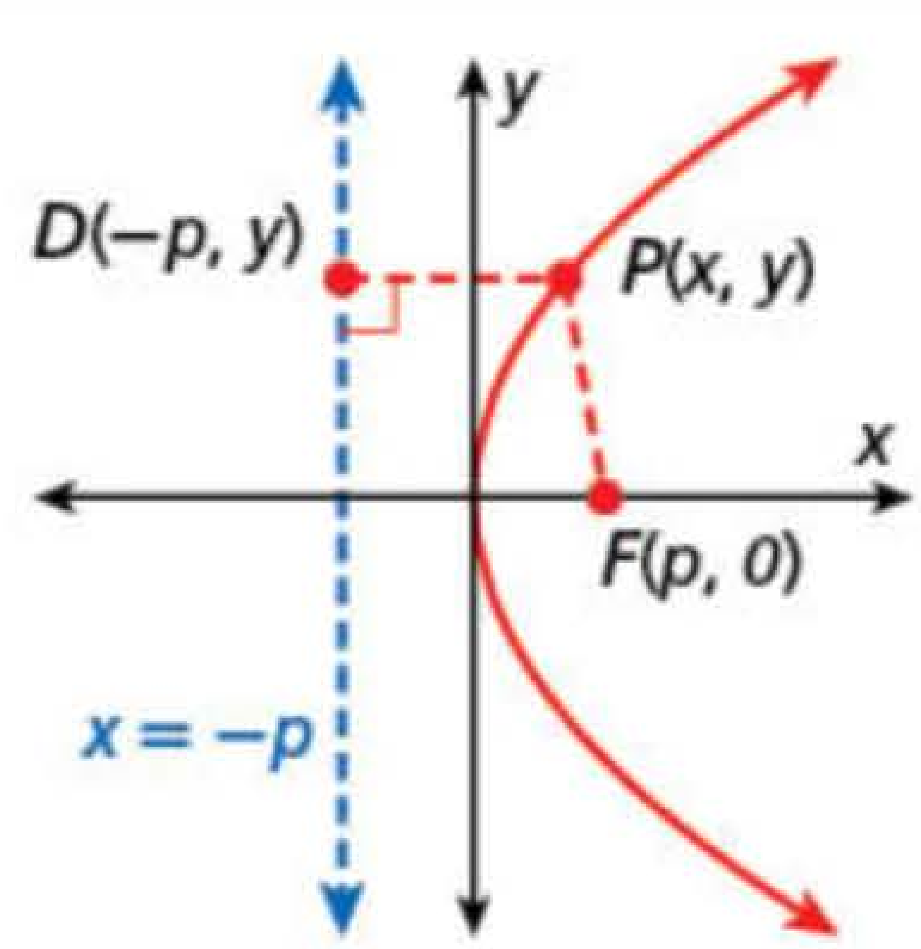
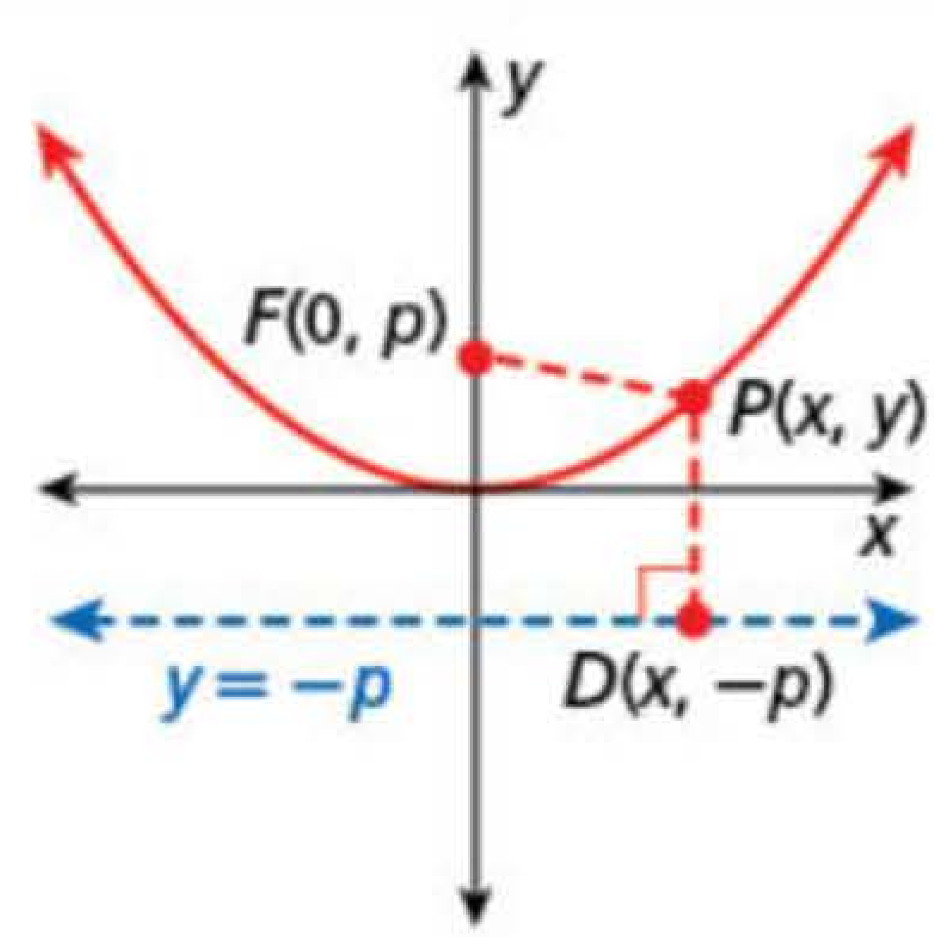
Try These Problems

Previously, you have graphed parabolas with vertical axes of symmetry that open upward or downward. Parabolas may also have horizontal axes of symmetry and may open to the left or right.

The equations of parabolas use the parameter $p$. The $|p|$ gives the distance from the vertex to both the focus and the directrix.

Know it! Note

### Standard Form for the Equation of a Parabola Vertex at (0, 0)

| AXIS OF SYMMETRY | HORIZONTAL $y = 0$ | VERTICAL $x = 0$ |
|---|---|---|
| Equation | $x = \frac{1}{4p}y^2$ | $y = \frac{1}{4p}x^2$ |
| Direction | Opens right if $p > 0$<br>Opens left if $p < 0$ | Opens upward if $p > 0$<br>Opens downward if $p < 0$ |
| Focus | $(p, 0)$ | $(0, p)$ |
| Directrix | $x = -p$ | $y = -p$ |
| Graph |   |   |

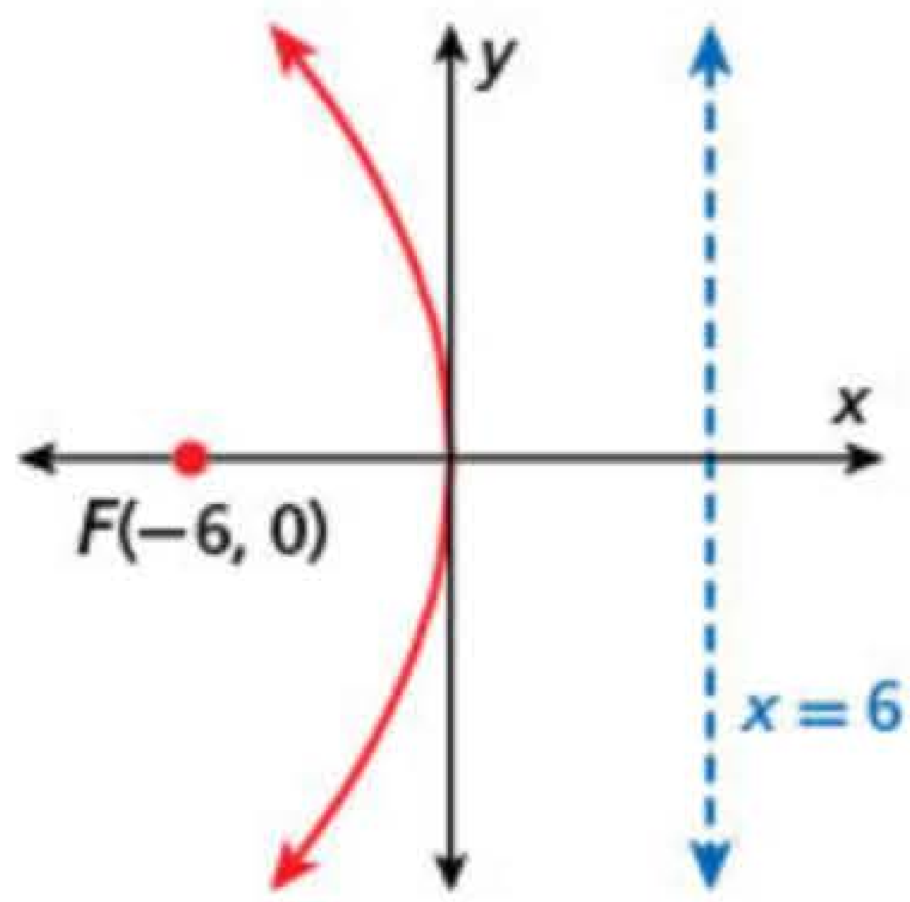
EXAMPLE 2

EXAMPLE 2

## Writing Equations of Parabolas

**Write the equation in standard form for each parabola.**

**A**

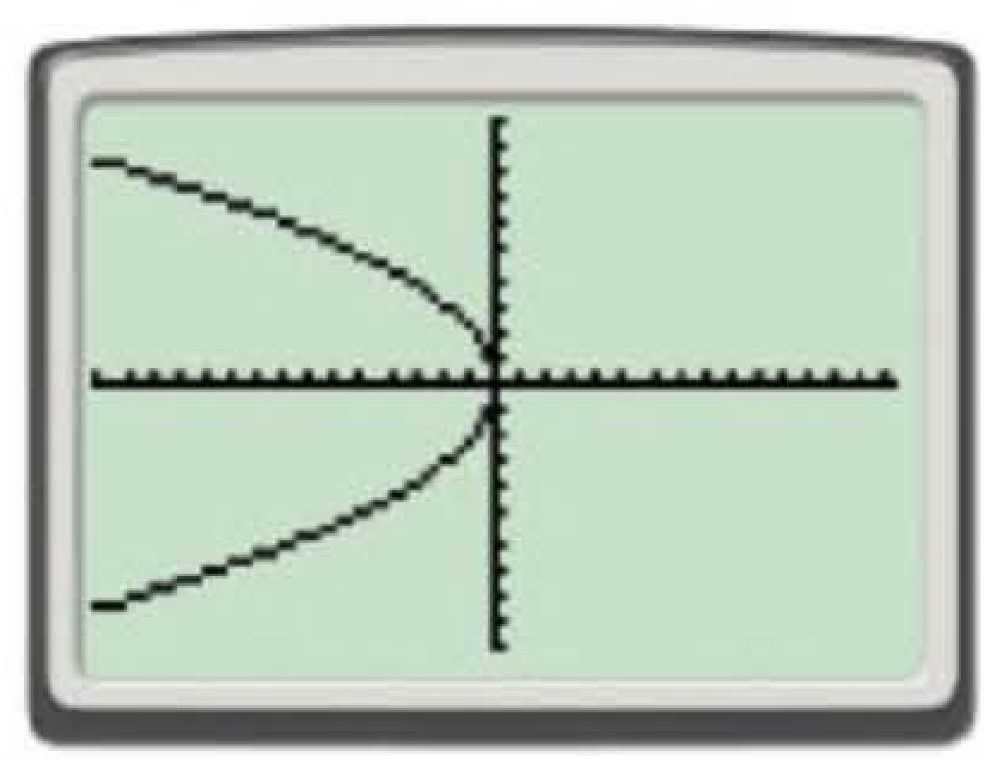



**Step 1** Because the axis of symmetry is horizontal and the parabola opens to the left, the equation is in the form $x = \frac{1}{4p}y^2$ with $p < 0$.

**Step 2** The distance from the focus $(-6, 0)$ to the vertex $(0, 0)$ is 6, so $p = -6$ and $4p = -24$.

**Step 3** The equation of the parabola is $x = -\frac{1}{24}y^2$.

*Check* Use your graphing calculator. The graph of the equation appears to match.



**Write the equation in standard form for each parabola.**

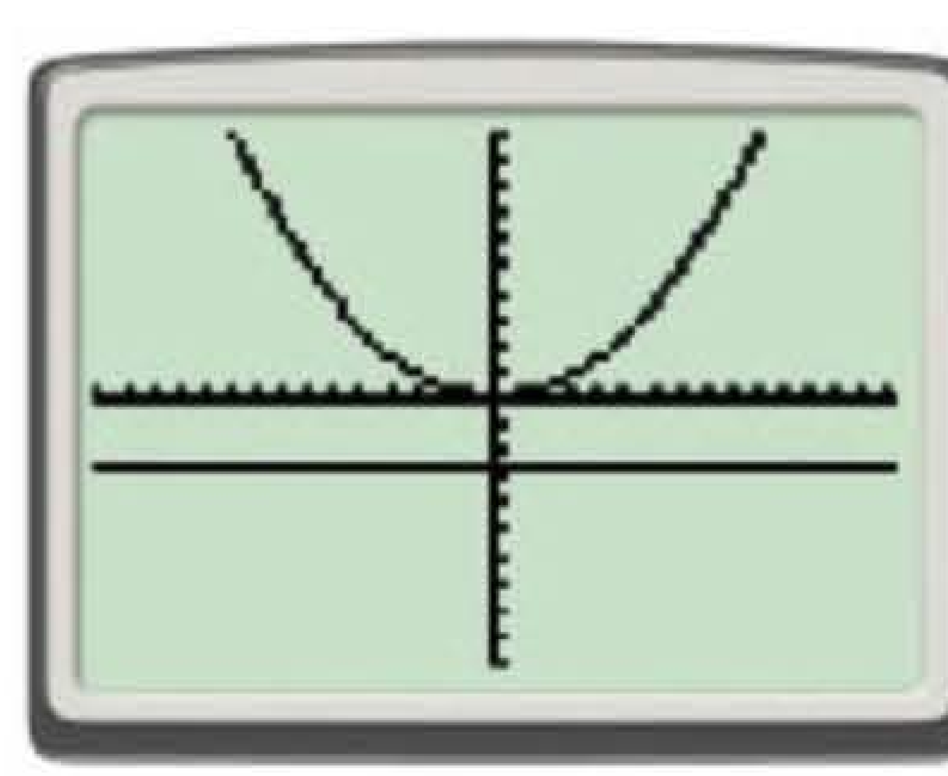
**B** the parabola with vertex $(0, 0)$ and directrix $y = -2.5$.

**Step 1** Because the directrix is a horizontal line, the equation is in the form $y = \frac{1}{4p}x^2$. The vertex is above the directrix, so the graph will open upward.

**Step 2** Because the directrix is $y = -2.5$, $p = 2.5$ and $4p = 10$.

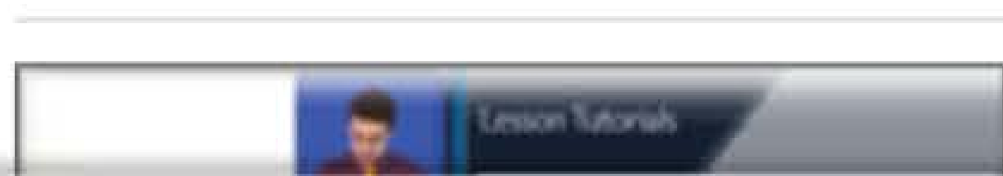
**Step 3** The equation of the parabola is $y = \frac{1}{10}x^2$.

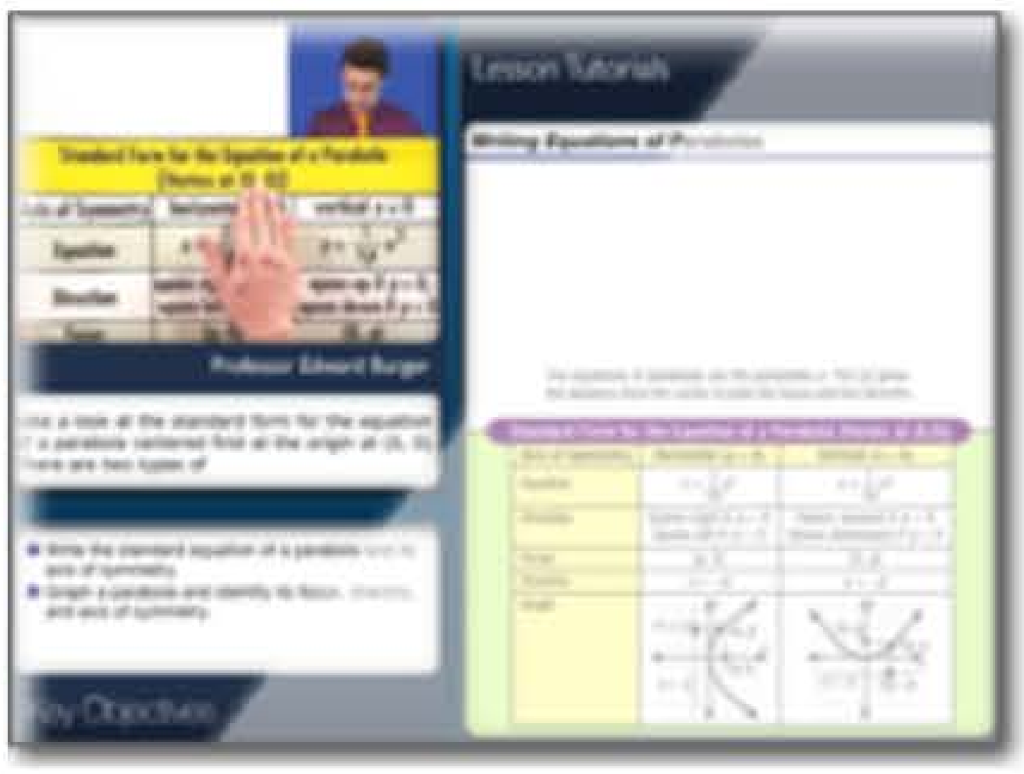
*Check* Use your graphing calculator.



Tap the button to view Example 2 in StepReveal.

View in Motion





Tap the button to watch the lesson tutorial video for extra instruction.

Play Video



## Check It Out!

**Write the equation in standard form for each parabola.**

**2a.** vertex $(0, 0)$, directrix $x = 1.25$

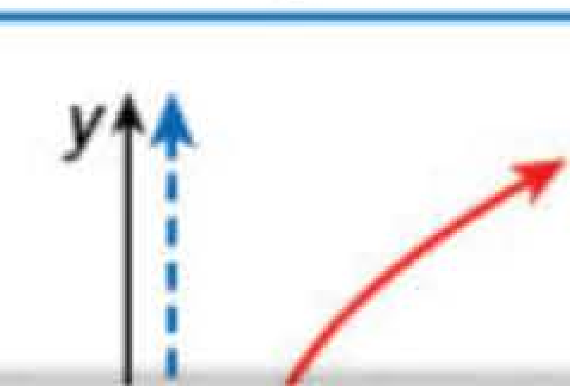
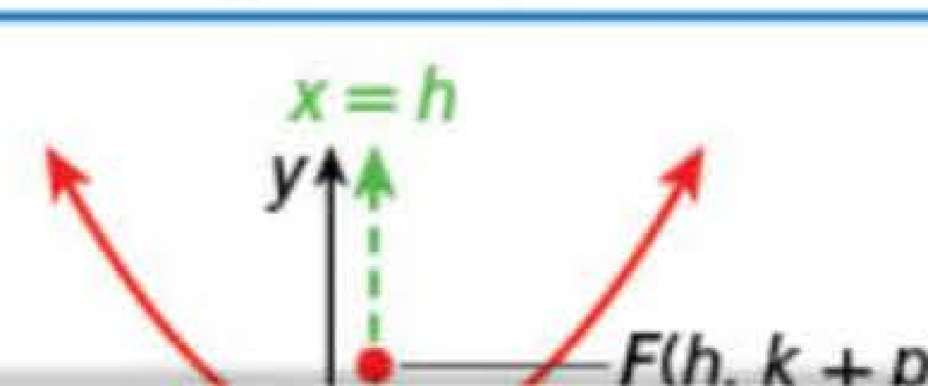
**2b.** vertex $(0, 0)$, focus $(0, -7)$

Try These Problems

The vertex of a parabola may not always be the origin. Adding or subtracting a value from *x* or *y* translates the graph of a parabola. Also notice that the values of *p* stretch or compress the graph.

Know it! Note

### Standard Form for the Equation of a Parabola Vertex at (h, k)

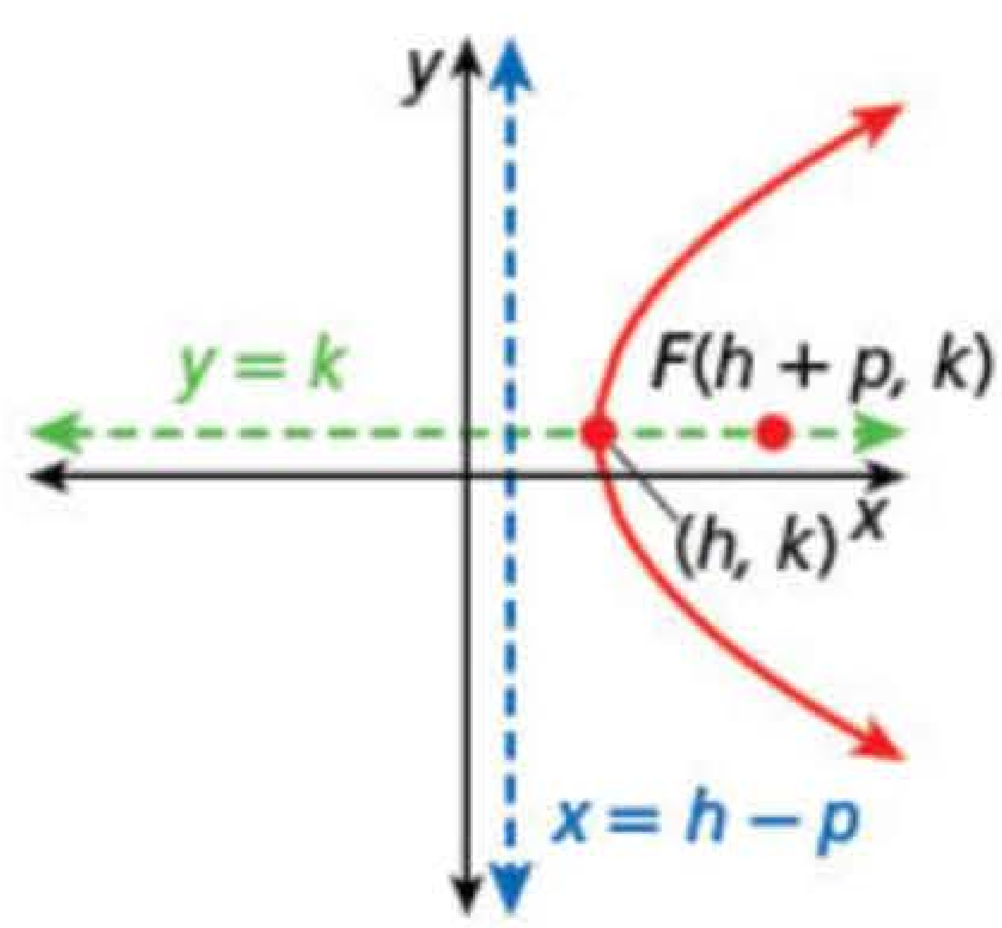
| AXIS OF SYMMETRY | HORIZONTAL $y = k$ | VERTICAL $x = h$ |
|---|---|---|
| Equation | $x - h = \frac{1}{4p}(y - k)^2$ | $y - k = \frac{1}{4p}(x - h)^2$ |
| Direction | Opens right if $p > 0$<br>Opens left if $p < 0$ | Opens upward if $p > 0$<br>Opens downward if $p < 0$ |
| Focus | $(h + p, k)$ | $(h, k + p)$ |
| Directrix | $x = h - p$ | $y = k - p$ |
| Graph |  |  |

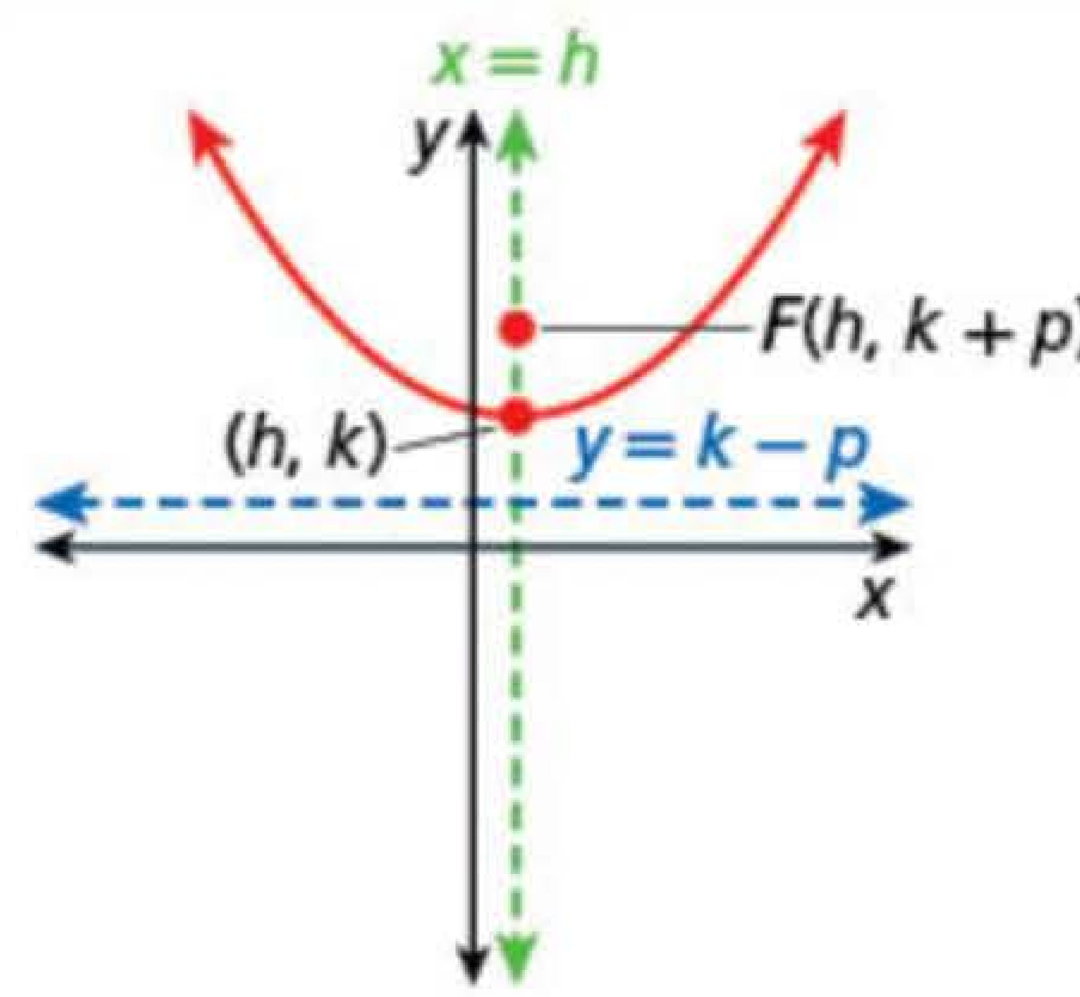
## Standard Form for the Equation of a Parabola Vertex at (h, k)

Know it! Note

| AXIS OF SYMMETRY | HORIZONTAL $y = k$ | VERTICAL $x = h$ |
|---|---|---|
| Equation | $x - h = \frac{1}{4p}(y - k)^2$ | $y - k = \frac{1}{4p}(x - h)^2$ |
| Direction | Opens right if $p > 0$<br>Opens left if $p < 0$ | Opens upward if $p > 0$<br>Opens downward if $p < 0$ |
| Focus | $(h + p, k)$ | $(h, k + p)$ |
| Directrix | $x = h - p$ | $y = k - p$ |

**Graph**




EXAMPLE 3

### Graphing Parabolas

**Find the vertex, value of $p$, axis of symmetry, focus, and directrix of the parabola $x - 2 = -\frac{1}{16}(y + 5)^2$. Then graph.**

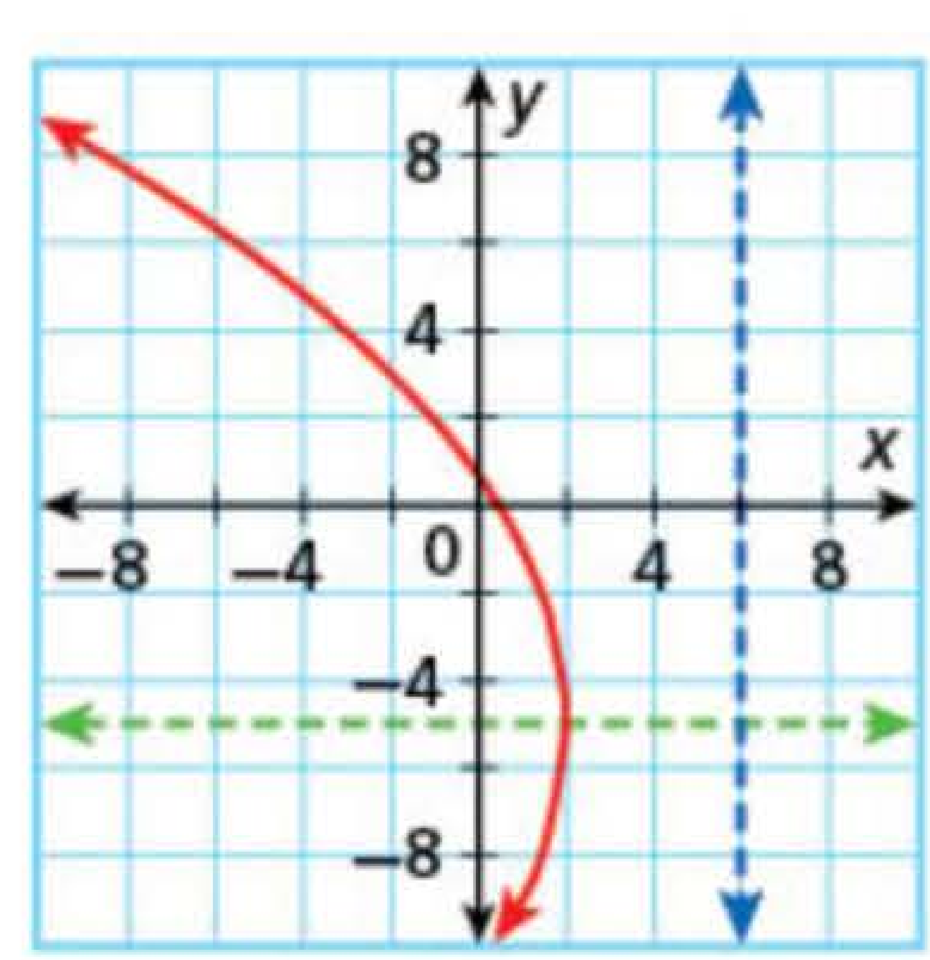
**Step 1** The vertex is $(2, -5)$.

**Step 2** $\frac{1}{4p} = -\frac{1}{16}$, so $4p = -16$ and $p = -4$.

**Step 3** The graph has a horizontal axis of symmetry, with equation $y = -5$, and opens left.

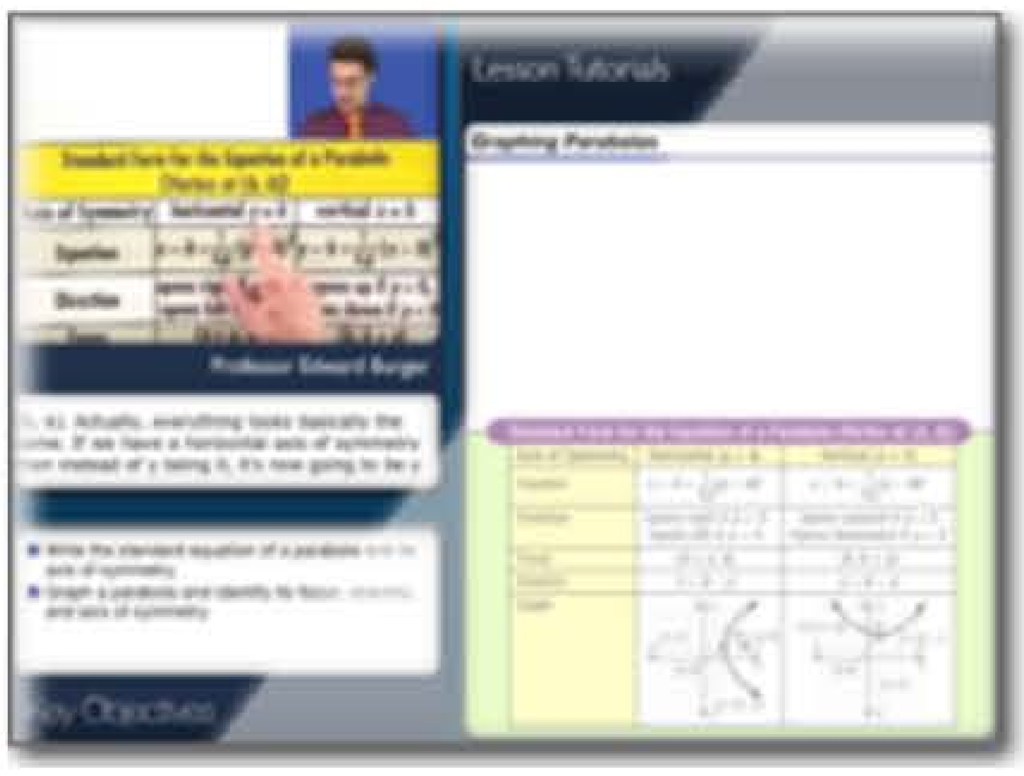
**Step 4** The focus is $(2 + (-4), -5)$, or $(-2, -5)$.

**Step 5** The directrix is a vertical line $x = 2 - (-4)$, or $x = 6$.



Tap the button to view Example 3 in StepReveal.

View in Motion




Tap the button to watch the lesson tutorial video for extra instruction.

Play Video



## Check It Out!

**Find the vertex, value of $p$, axis of symmetry, focus, and directrix of each parabola. Then graph.**

**3a.** $x - 1 = \frac{1}{12}(y - 3)^2$

**3b.** $y - 4 = -\frac{1}{2}(x - 8)^2$

Try These Problems

Light or sound waves collected by a parabola will be reflected by the curve through the focus of the parabola, as shown in the figure. Waves emitted from the focus will be reflected out parallel to the axis of symmetry of a parabola. This property is used in communications technology.

**EXAMPLE 4**

### Using the Equation of a Parabola

Engineers are constructing a parabolic microphone for use at sporting events. The surface of the parabolic microphone will reflect sounds to the focus of the microphone at the end of a part called a feedhorn. The equation for the cross section of the parabolic

EXAMPLE 4

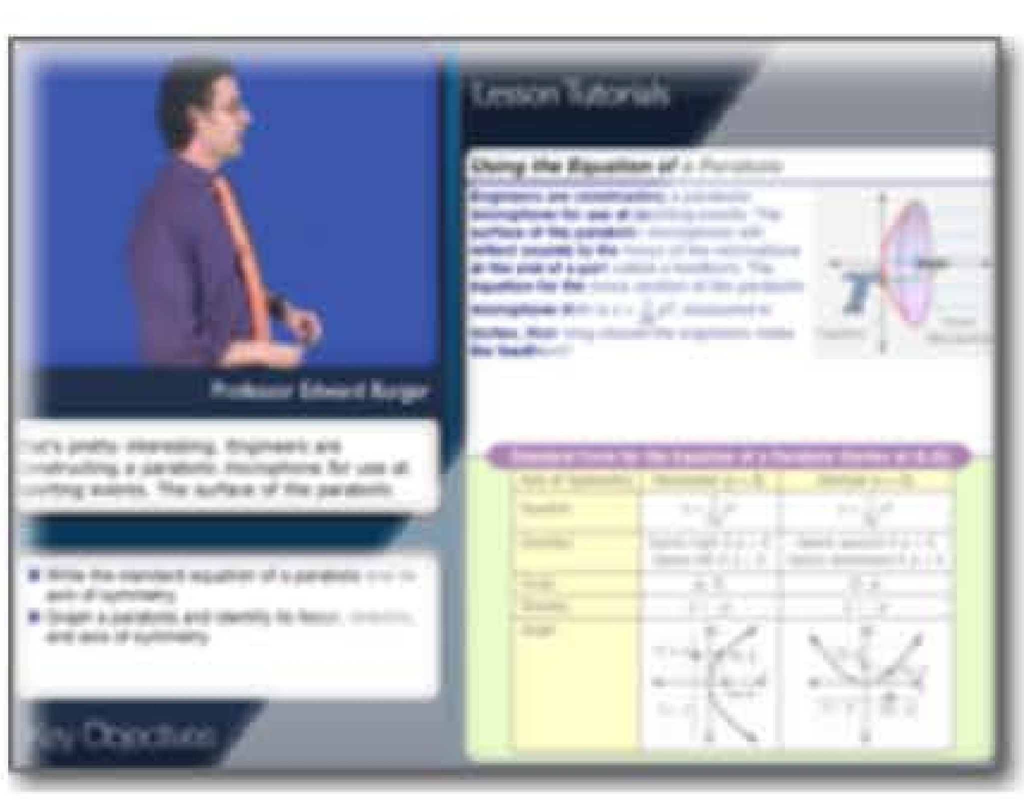
## Using the Equation of a Parabola

Engineers are constructing a parabolic microphone for use at sporting events. The surface of the parabolic microphone will reflect sounds to the focus of the microphone at the end of a part called a feedhorn. The equation for the cross section of the parabolic microphone dish is $x = \frac{1}{32}y^2$, measured in inches. How long should the engineers make the feedhorn?


The equation for the cross section is in the form $x = \frac{1}{4p}y^2$, so $4p = 32$ and $p = 8$. The focus should be 8 inches from the vertex of the cross section. Therefore, the feedhorn should be 8 inches long.

Tap the button to view Example 4 in StepReveal.

View in Motion



Tap the button to watch the lesson tutorial video for extra instruction.

Play Video

## Check It Out!

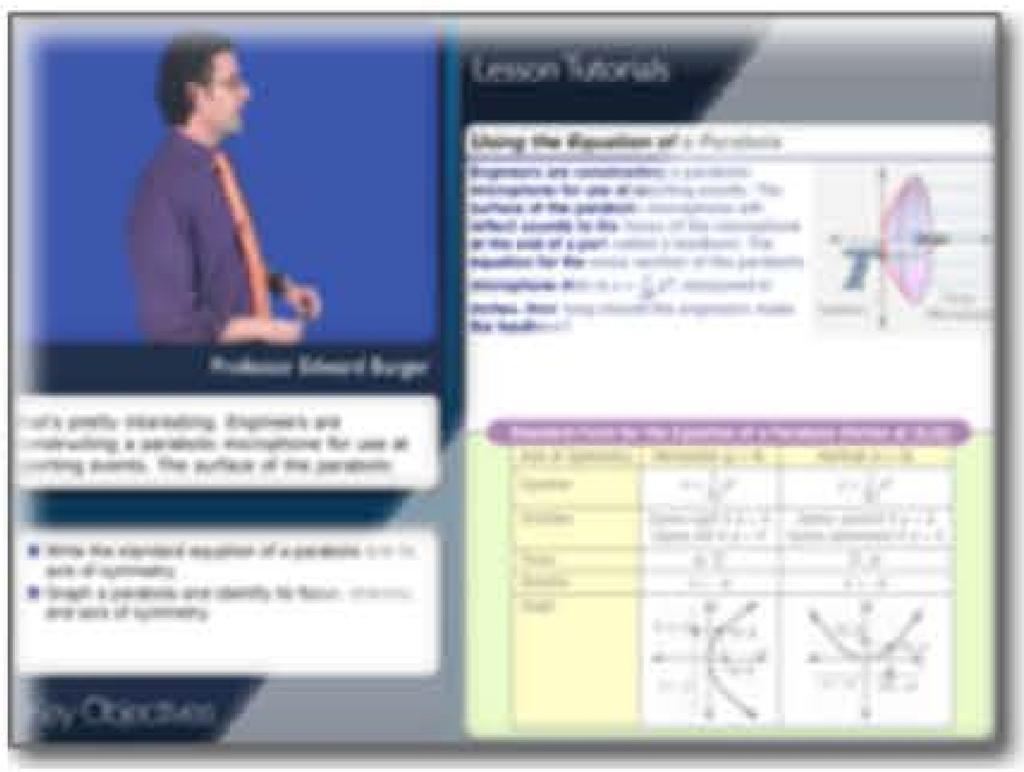
**4.** Find the length of the feedhorn for a microphone with a cross section equation $x = \frac{1}{44}y^2$.

Try These Problems

the feedhorn should be 8 inches long.

Tap the button to view Example 4 in StepReveal.

View in Motion




Tap the button to watch the lesson tutorial video for extra instruction.

Play Video

## Check It Out!

**4.** Find the length of the feedhorn for a microphone with a cross section equation $x = \frac{1}{44}y^2$.

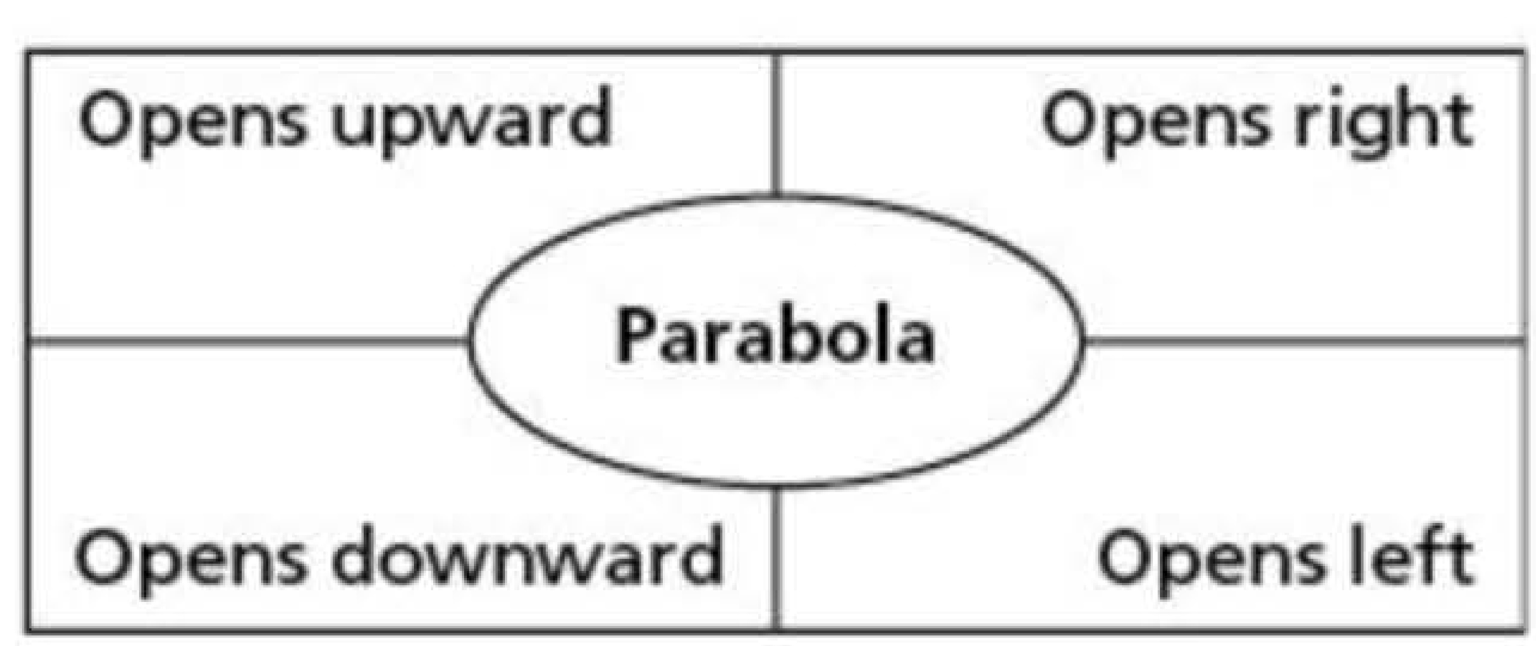
Try These Problems

## THINK AND DISCUSS

**1.** By using the standard form of a parabola's equation, how can you tell which direction a parabola opens?

**2.** How does knowing the value of $p$ help you in finding the focus and the directrix of a parabola?

**3. GET ORGANIZED** Copy and complete the graphic organizer. Sketch an example and give an equation for each type of parabola.




Know it! Note

# GUIDED PRACTICE

1. **Vocabulary** Describe the relationship between a parabola and its *directrix*.

**Use the distance formula to find the equation of a parabola with the given focus and directrix.**

see example 1

2. $F(0, -5), y = 5$

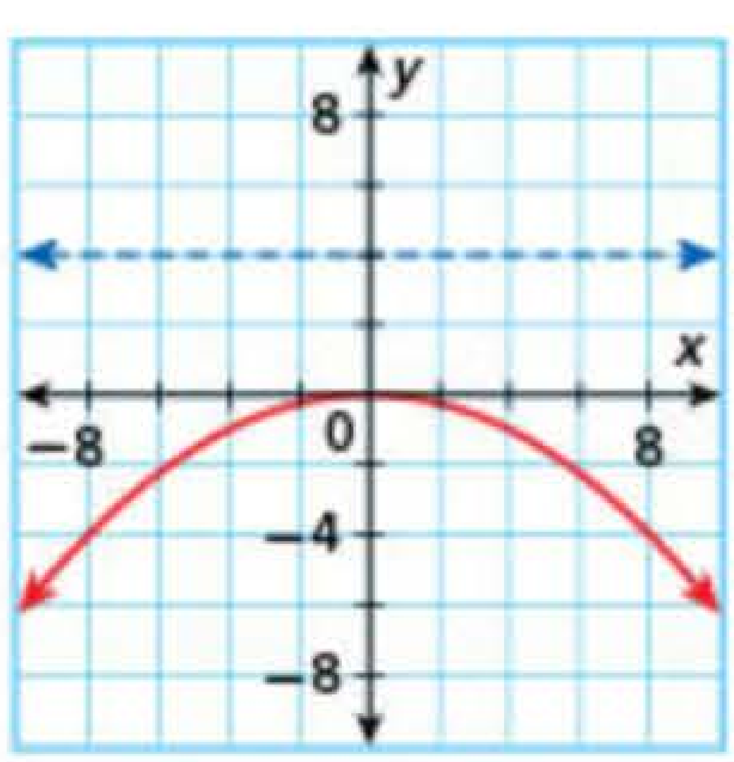
3. $F(7, 0), x = -7$

4. $F(-3, 0), x = 6$

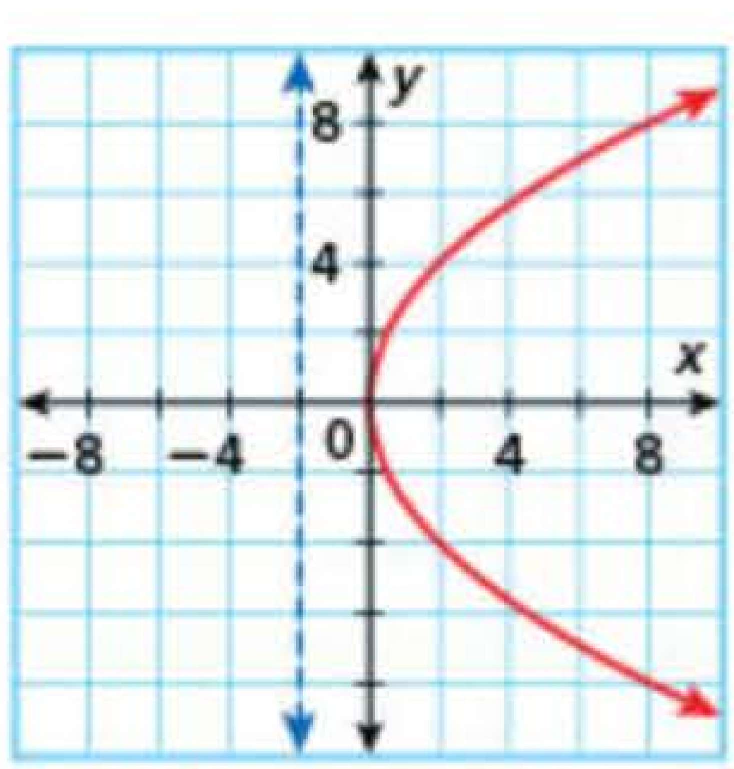
**Write the equation in standard form for each parabola.**

see example 2

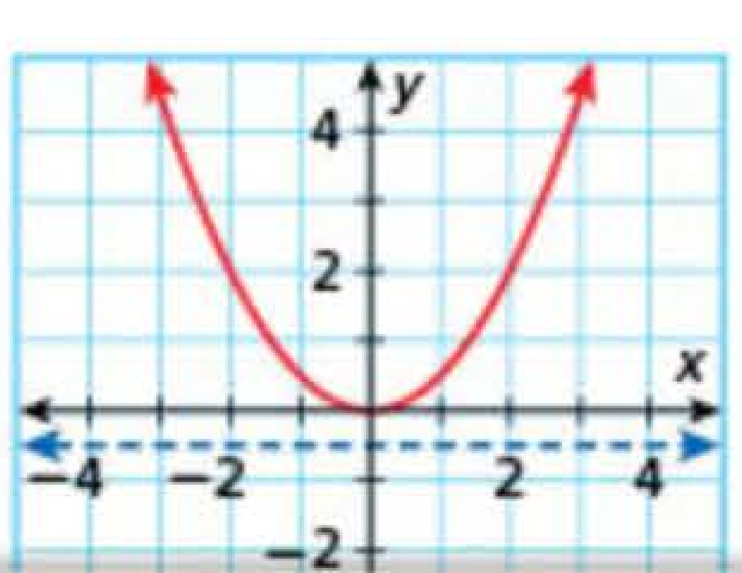
5.

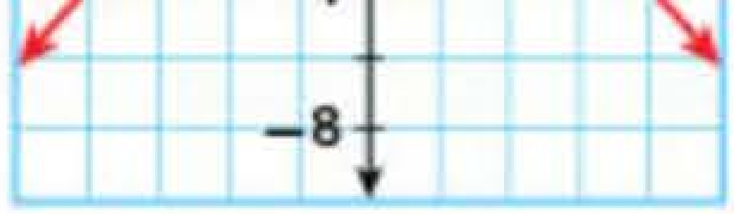



6.

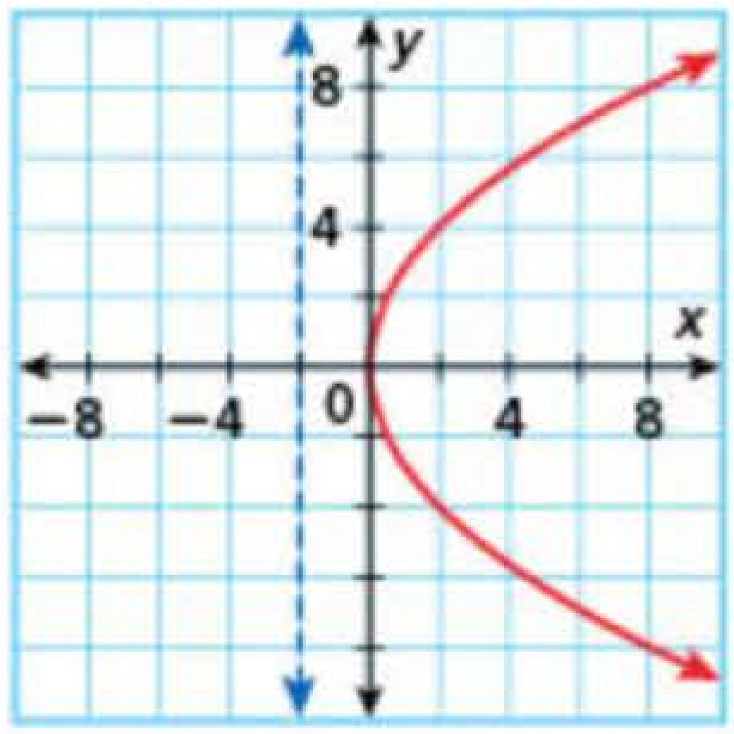



7.

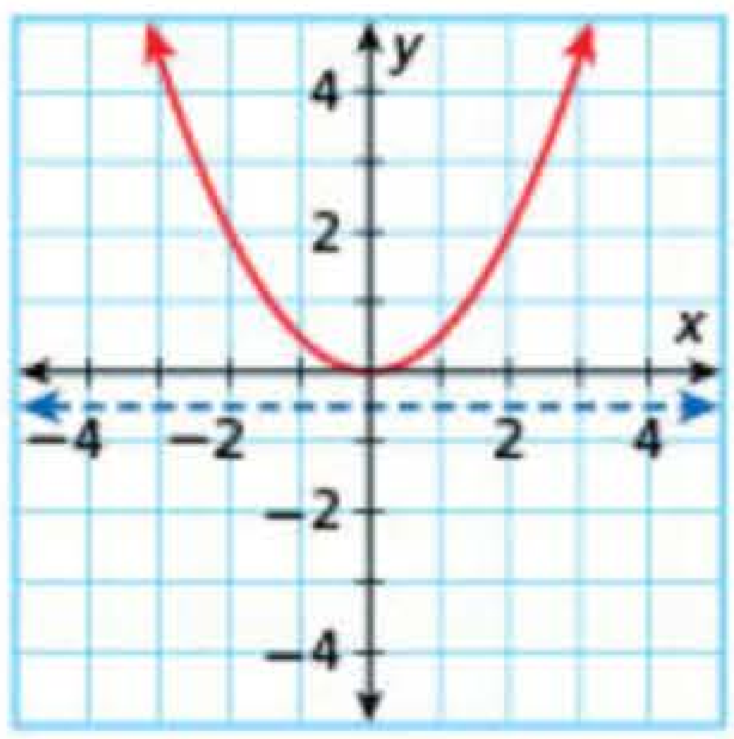





**6**




**7**




**8** vertex $(0, 0)$, focus $(0, 1)$

**9** vertex $(0, 0)$, focus $(-8, 0)$

**Find the vertex, value of p, axis of symmetry, focus, and directrix of each parabola, and then graph.**

see example 3

**10** $y = \frac{1}{32}(x + 2)^2$

**11** $x = \frac{1}{24}(y - 4)^2$

**12** $y + 1 = \frac{1}{16}(x - 2)^2$

see example 4

**13**

**Communications** The equation for the cross section of a parabolic satellite TV dish is $y = \frac{1}{38}x^2$, measured in inches. How far is the focus from the vertex of the cross section?

# PRACTICE AND PROBLEM SOLVING

852

**MULTI-STEP TEST PREP**

An opportunity to investigate multi-step real-world algebra based problem

View

Independent Practice | Skills Practice | Application Practice

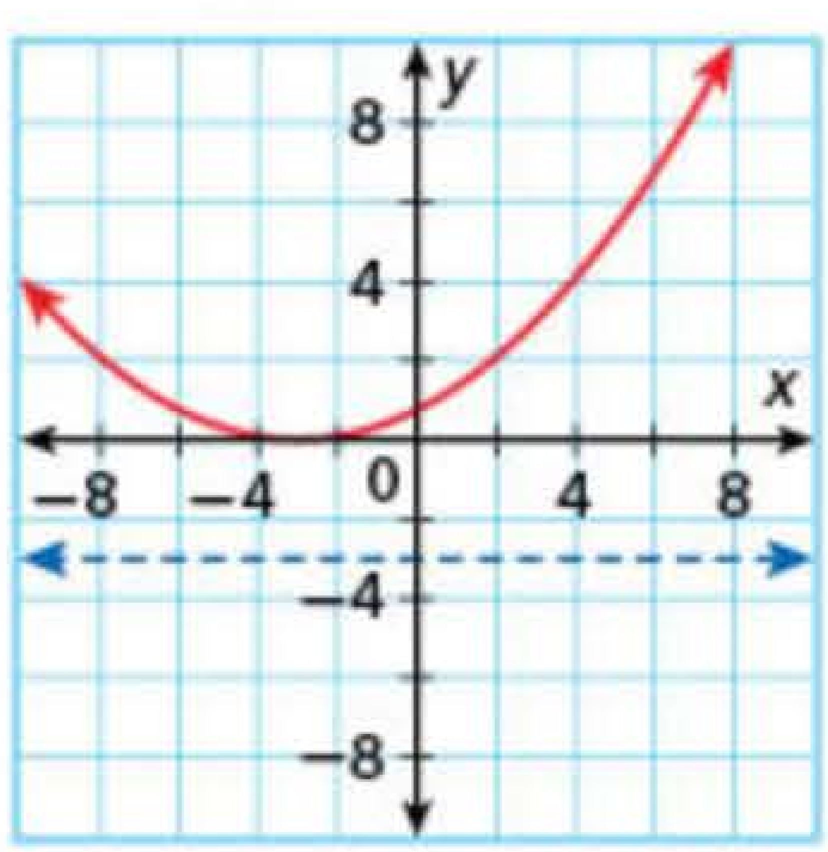
Homework Help
#27
#29
#31
#33
#35

**Use the distance formula to find the equation of a parabola with the given focus and directrix.**

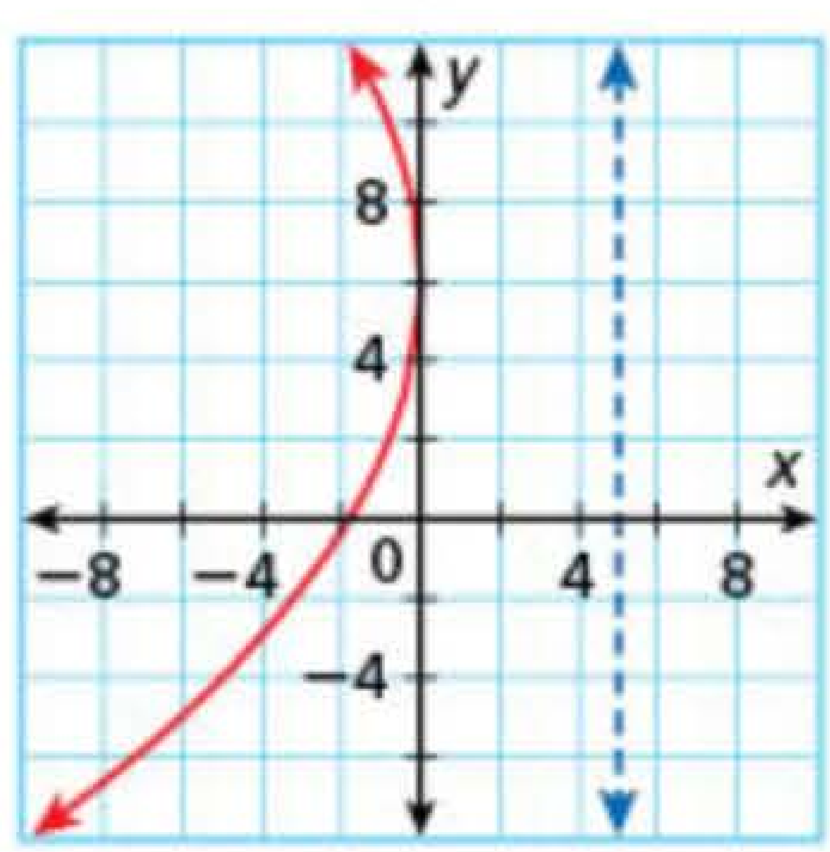
**14.** $F(0, 3), y = -5$ **15.** $F(-2, 0), x = 8$ **16.** $F(7, 0), x = -1$

**Write the equation in standard form for each parabola.**

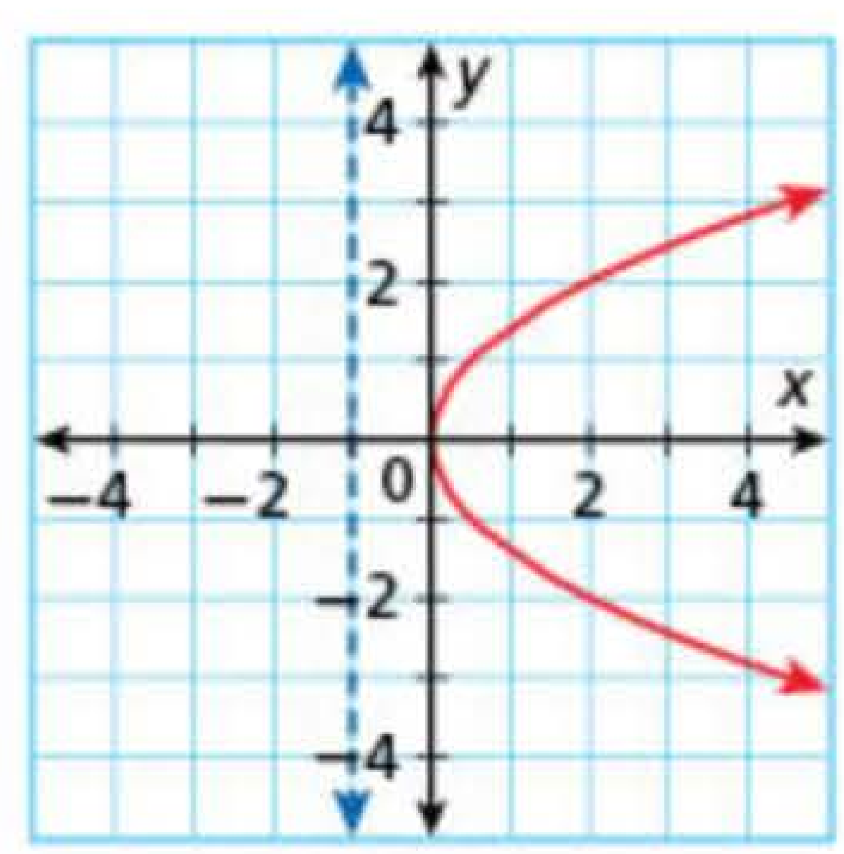
**17.**



**18.**



**19.**



**20.** vertex $(0, 0)$, focus $\left(\frac{1}{2}, 0\right)$ **21.** vertex $(0, 0)$, focus $(0, -6)$

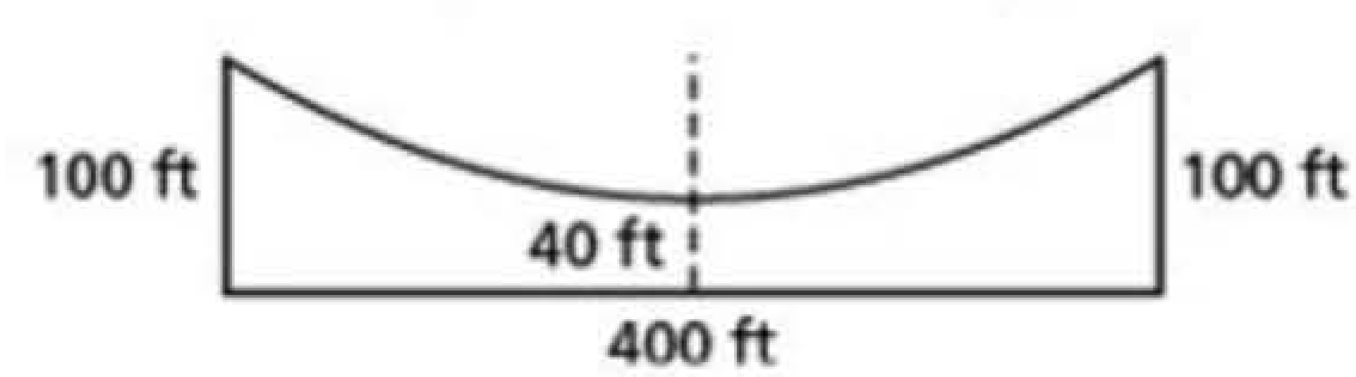
**Find the vertex, value of $p$, axis of symmetry, focus, and directrix of each parabola, and then graph.**

**22.** $y = \frac{1}{8}(x-1)^2$ **23.** $x = 2y^2 + 1$ **24.** $x - 2 = \frac{1}{2}(y+1)^2$

850

**25.** **Communications** Find an equation for a cross section of a parabolic microphone whose feedhorn is 9 inches long if the end of the feedhorn is placed at the origin.

**26.** **Engineering** The main cables of a suspension bridge are ideally parabolic. The cables over a bridge that is 400 feet long are attached to towers that are 100 feet tall. The lowest point of the cable is 40 feet above the bridge.




**a.** Find the coordinates of the vertex and the tops of the towers if the bridge represents the $x$-axis and the axis of symmetry is the $y$-axis.

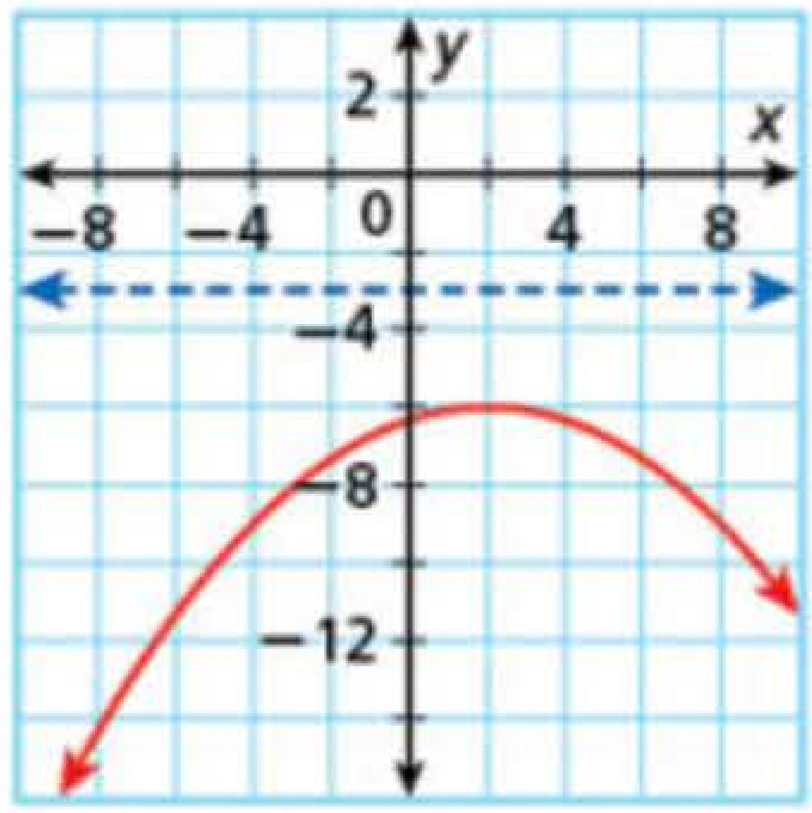
**b.** Find an equation that can be used to model the cables.

Link

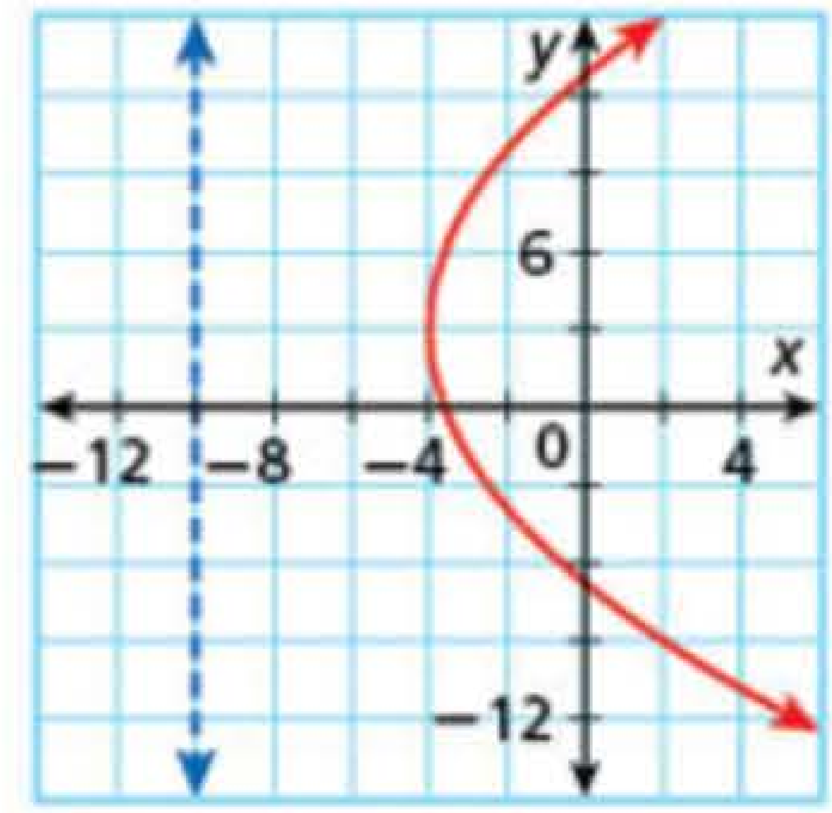
**Write the equation in standard form for each parabola, and give the domain and range.** (*Hint:* Find the domain and range by using the vertex and the direction that the parabola opens.)

Write the equation in standard form for each parabola, and give the domain and range. (*Hint:* Find the domain and range by using the vertex and the direction that the parabola opens.)

**27.**




**28.**




**29.** vertex $(-7, -3)$, focus $(2, -3)$

**30.** vertex $(5, -2)$, focus $(5, -8)$

**31.** focus $(0, 0)$, directrix $y = 10$

**32.** focus $(2, 6)$, directrix $y = -8$

**33.** focus $(4, -5)$, directrix $x = 12$

**34.** focus $(-3, 1)$, directrix $x = -15$

**35. Engineering** A spotlight has parabolic cross sections.

a. Write an equation for a cross section of the spotlight if the bulb is 5 inches from the vertex and the vertex is placed at the origin.

b. Write an equation for a cross section of the spotlight if the bulb is 4 inches from the vertex and the bulb is placed at the origin.

c. If the spotlight has a diameter of 24 inches at its opening, find the depth of the spotlight if the bulb is 5 inches from the vertex.

**36. Sports** When a football is kicked, the path that the ball travels can be modeled by a parabola.

a. A placekicker kicks a football, which reaches a maximum height of 8 yards and lands 50 yards away. Assuming that the football was at the origin when it was kicked, write an equation for the height of the football.

b. **What if...?** If the placekicker was trying to kick the ball over a 10-foot-high goalpost 40 yards away, was the football high enough to go over the goalpost? Explain.

**MULTI-STEP TEST PREP**

**37.** The path of a comet is modeled by the parabola $y = -\frac{1}{532}(x + 96)^2 + 174$, where each unit of the coordinate plane represents 1 million kilometers.

a. The Sun is at the focus of the parabolic path. Find the coordinates of the Sun.

b. How close does the comet come to the Sun?

c. What are the coordinates of the comet when it is at its closest point to the Sun?

Graph each equation. Identify the vertex, value of $p$, axis of symmetry, focus, and directrix for each equation.

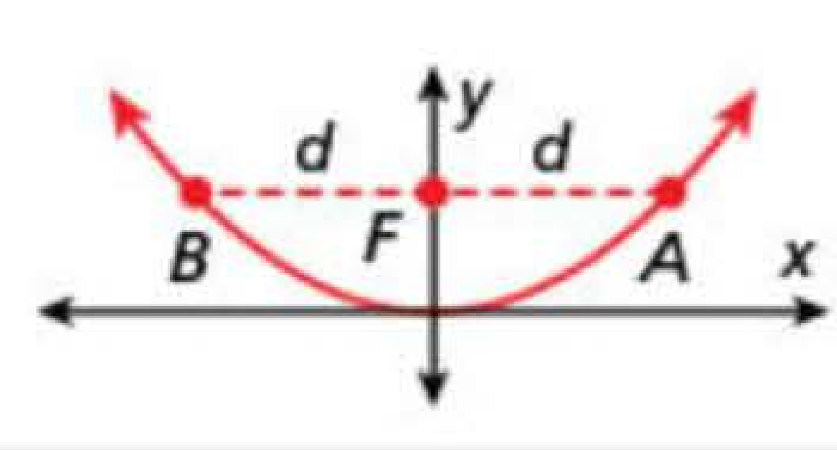
**38.** $20(y - 2) = (x + 6)^2$

**39.** $y = -2(x + 4)^2 + 5$

**40.** $(y + 7)^2 = \frac{x}{16}$

**41.** $x + 3 = \frac{1}{8}(y - 2)^2$

**42. Critical Thinking** Find the distance $d$ from the focus to the points on the parabola that are on the line perpendicular to the axis of symmetry and through the focus. Explain your answer.



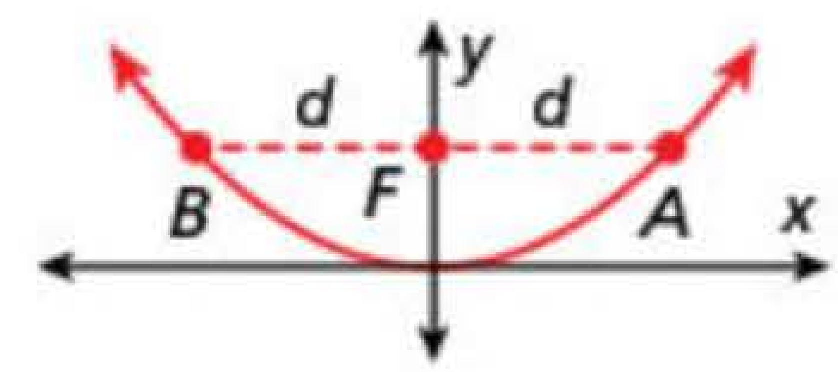
38. $20(y-2)=(x+6)^2$

39. $y=-2(x+4)^2+5$

40. $(y+7)^2=\frac{x}{16}$

41. $x+3=\frac{1}{8}(y-2)^2$

42. **Critical Thinking** Find the distance $d$ from the focus to the points on the parabola that are on the line perpendicular to the axis of symmetry and through the focus. Explain your answer.




43. **Write About It** Explain how changing the value of $p$ will affect the vertex, focus, and directrix of the parabola $y-k=\frac{1}{4p}(x-h)^2$.

## Test Prep

44. The graph of which of the following parabolas opens to the left?

Ⓐ $16y-4x^2=12$ Ⓑ $16y+4x^2=12$ Ⓒ $16x-4y^2=12$ Ⓓ $16x+4y^2=12$

45. Which of the following is the axis of symmetry for the graph of $x-4=\frac{1}{8}(y+2)^2$?

Ⓕ $x=0$ Ⓖ $y=-2$ Ⓗ $x=4$ Ⓙ $y=8$

46. Which of the following graphs has the directrix $y=4$?

Ⓐ $y+3=\frac{1}{4}(x-1)^2$

Ⓑ $y-5=\frac{1}{4}(x+2)^2$

Ⓒ $x-5=\frac{1}{4}(y+4)^2$

Ⓓ $x+3=\frac{1}{4}(y-2)^2$

47. **Short Response** What are the coordinates of the focus for the graph of $x-3=\frac{1}{16}y^2$?

Check Answers

## CHALLENGE AND EXTEND

**Write the equation in standard form for each parabola.**

48. vertex $(6, 8)$, contains the point $(4, -2)$, axis of symmetry $x=6$

49. focus $(6, 5)$, axis of symmetry $x=6$, contains the point $(10, 5)$

**Multi-Step** **The latus rectum of a parabola is the line segment perpendicular to the axis of symmetry through the focus, with endpoints on the parabola. Find the length of the latus rectum of each parabola.**

50. $y=\frac{1}{8}x^2$

51. $y-k=\frac{1}{4p}(x-h)^2$

Check Answers

# Identifying Conic Sections

861

TECHNOLOGY LAB

A technology based activity which reinforces the chapter concepts

View

854

## Why learn this?

The path of an airplane in a dive can be modeled by a branch of a hyperbola or a parabola. (See Example 4.)

## Lesson Objective(s):

- Identify and transform conic sections.
- Use the method of completing the square to identify and graph conic sections.

Previously, you have learned about the four conic sections. Recall the equations of conic sections in standard form. In these forms, the characteristics of the conic sections can be identified.

**Standard Forms for the Conic Sections with Center (h, k)**

| | | |
|---|---|---|
| **Circle** | $(x-h)^2+(y-k)^2=r^2$ | |
| | **HORIZONTAL AXIS** | **VERTICAL AXIS** |
| **Ellipse** | $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ | $\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$ |
| **Hyperbola** | $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$ | $\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$ |
| **Parabola** | $x-h=\frac{1}{4p}(y-k)^2$ | $y-k=\frac{1}{4p}(x-h)^2$ |

Know it! Note

EXAMPLE 1

## Identifying Conic Sections in Standard Form

**Identify the conic section that each equation represents.**

**A** $\dfrac{(x-7)^2}{5^2} - \dfrac{(y+2)^2}{2^2} = 1$

This equation is of the same form as a hyperbola with a horizontal transverse axis.

**B** $y - 3 = \dfrac{1}{12}(x-4)^2$

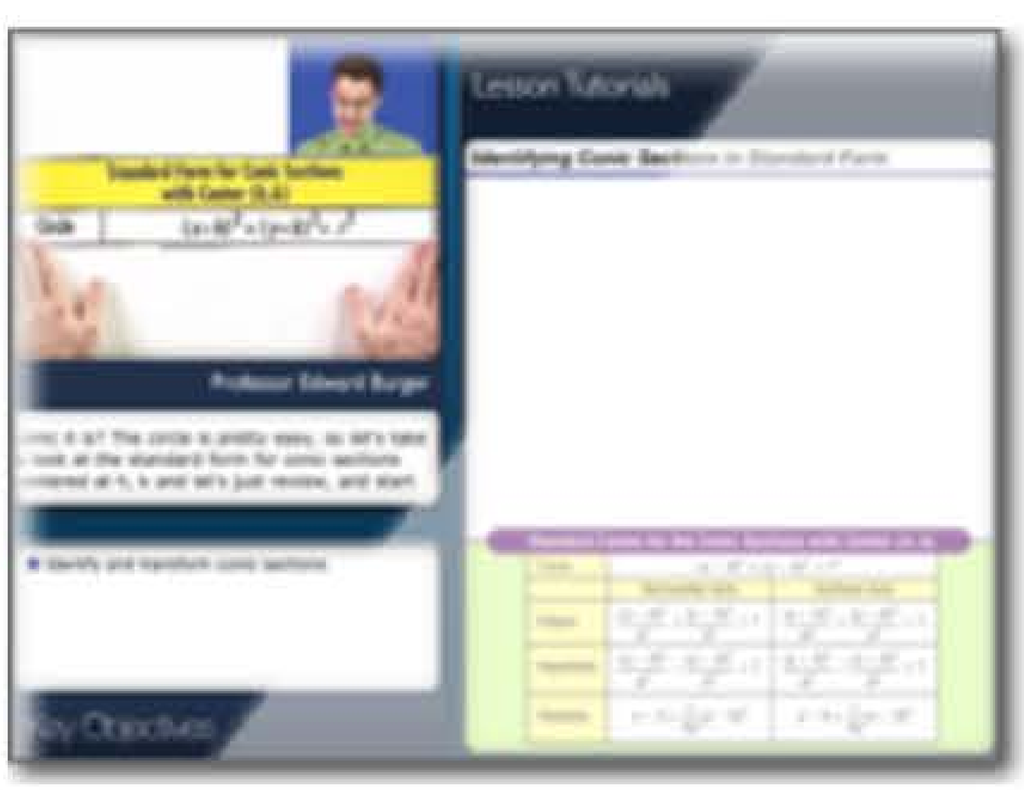
This equation is of the same form as a parabola with a vertical axis of symmetry.

**C** $\dfrac{(x-1)^2}{8^2} + \dfrac{(y-1)^2}{10^2} = 1$

This equation is of the same form as an ellipse with a vertical major axis.

Tap the button to view Example 1 in StepReveal.

View in Motion




Tap the button to watch the lesson tutorial video for extra instruction.

Play Video

## Check It Out!

**Identify the conic section that each equation represents.**

**1a.** $x^2 + (y+14)^2 = 11^2$

**1b.** $\dfrac{(y-6)^2}{2^2} - \dfrac{(x-1)^2}{21^2} = 1$

Try These Problems

## Student to Student

### Classifying Conic Sections

Mercedes Raya
Central High School

*I can classify an equation in standard form just by looking. This is a good way for me to check my work.*

*Only one squared term* → *It's a parabola.*

*A squared term minus a squared term* → *It's a hyperbola.*

*A squared term plus a squared term* → *It's a circle or an ellipse.*

All conic sections can be written in the general form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. The conic section represented by an equation in general form can be determined by the coefficients.

### Classifying Conic Sections

For an equation of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ ($A$, $B$, and $C$ do not all equal 0.)

| CONIC SECTION | COEFFICIENTS |
|---|---|
| Circle | $B^2 - 4AC < 0$, $B = 0$, and $A = C$ |
| Ellipse | $B^2 - 4AC < 0$ and either $B \neq 0$ or $A \neq C$ |
| Hyperbola | $B^2 - 4AC > 0$ |
| Parabola | $B^2 - 4AC = 0$ |

Know it! Note

**EXAMPLE 2**

## Identifying Conic Sections in General Form

**Identify the conic section that each equation represents.**

**A** $6x^2 + 9y^2 + 12x - 15y - 25 = 0$

$A = 6, B = 0, C = 9$ — *Identify the values for A, B, and C.*

$B^2 - 4AC$

$0^2 - 4(6)(9)$ — *Substitute into $B^2 - 4AC$.*

$-216$ — *Simplify. The conic is either a circle or an ellipse.*

$A \neq C$ — *The conic is not a circle.*

Because $B^2 - 4AC < 0$ and $A \neq C$, the equation represents an ellipse.

**B** $4x^2 + 4xy + y^2 - 12x + 8y + 36 = 0$

$A = 4, B = 4, C = 1$ — *Identify the values for A, B, and C.*

$B^2 - 4AC$

EXAMPLE 2

## Identifying Conic Sections in General Form

**Identify the conic section that each equation represents.**

**A** $6x^2 + 9y^2 + 12x - 15y - 25 = 0$

$A = 6, B = 0, C = 9$ — *Identify the values for A, B, and C.*

$B^2 - 4AC$

$0^2 - 4(6)(9)$ — *Substitute into $B^2 - 4AC$.*

$-216$ — *Simplify. The conic is either a circle or an ellipse.*

$A \neq C$ — *The conic is not a circle.*

Because $B^2 - 4AC < 0$ and $A \neq C$, the equation represents an ellipse.

**B** $4x^2 + 4xy + y^2 - 12x + 8y + 36 = 0$

$A = 4, B = 4, C = 1$ — *Identify the values for A, B, and C.*

$B^2 - 4AC$

$4^2 - 4(4)(1)$ — *Substitute into $B^2 - 4AC$.*

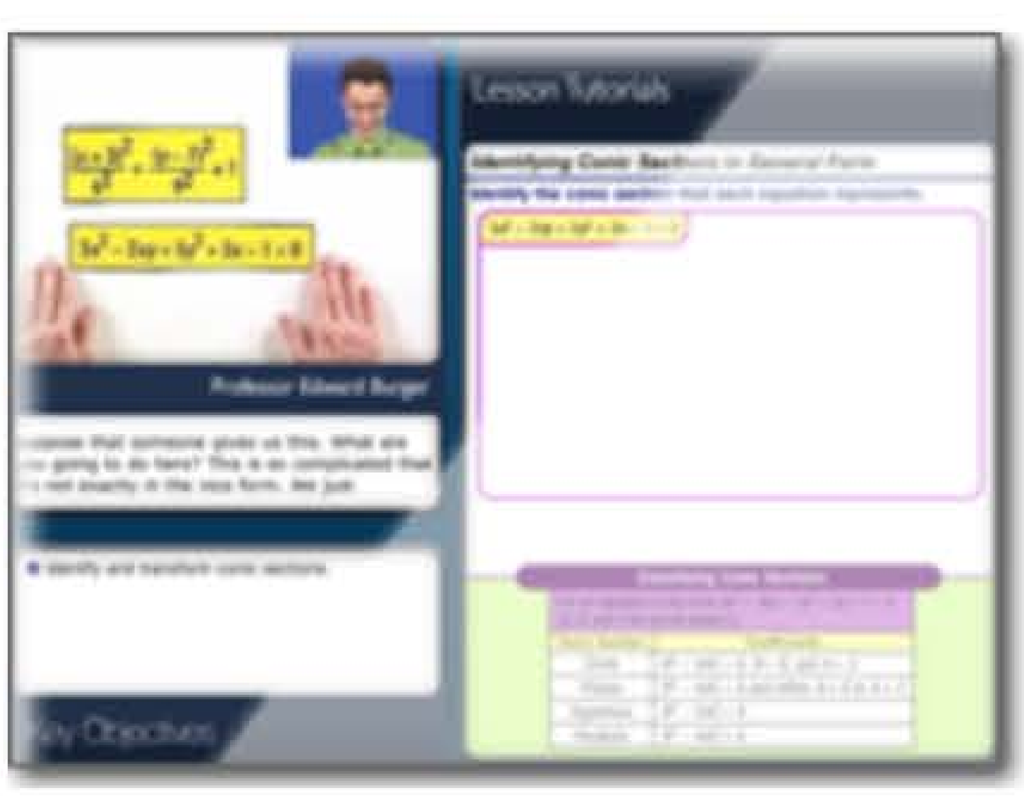
$0$ — *Simplify.*

Because $B^2 - 4AC = 0$, the equation represents a parabola.

---

Tap the button to view Example 2 in StepReveal. View in Motion

---




Tap the button to watch the lesson tutorial video for extra instruction. Play Video

## Check It Out!

**Identify the conic section that each equation represents.**

**2a.** $9x^2 + 9y^2 - 18x - 12y - 50 = 0$

**2b.** $12x^2 + 24xy + 12y^2 + 25y = 0$

If you are given the equation of a conic in standard form, you can write the equation in general form by expanding the binomials.

If you are given the general form of a conic section, you can use the method of completing the square to write the equation in standard form.

**EXAMPLE 3**

## Finding the Standard Form of the Equation for a Conic Section

Remember

**Find the standard form of each equation by completing the square. Then identify and graph each conic.**

**A** $x^2 - 12x - 16y + 36 = 0$

$$x^2 - 12x + \square = 16y - 36 + \square$$ *Prepare to complete the square in x.*

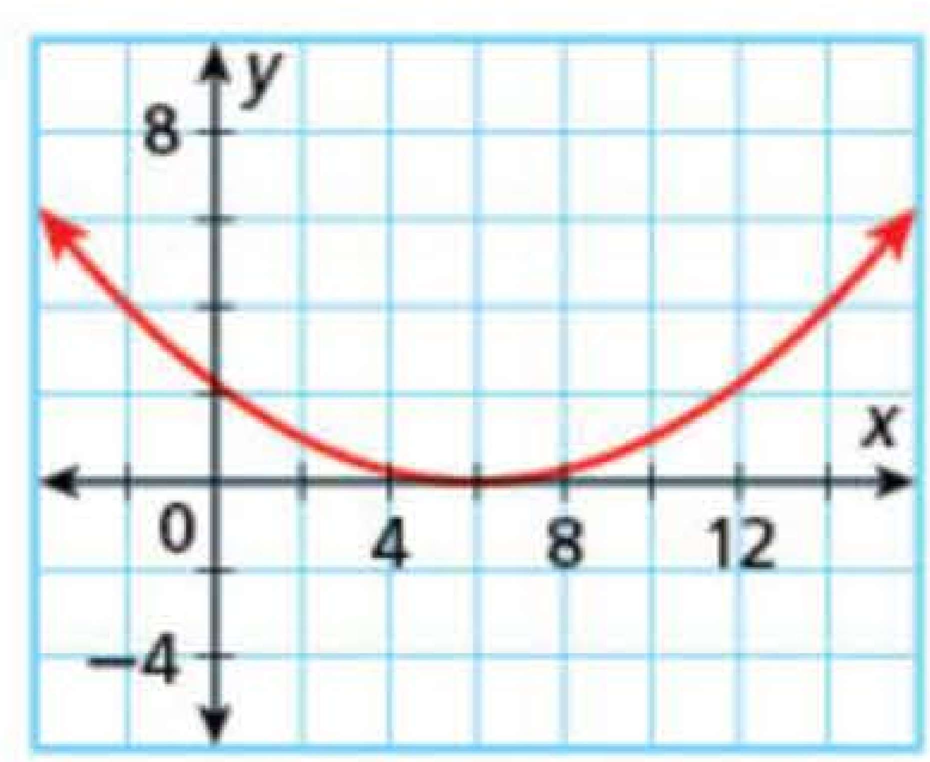
$$x^2 - 12x + \left(\frac{-12}{2}\right)^2 = 16y - 36 + \left(\frac{-12}{2}\right)^2$$ *Add $\left(-\frac{12}{2}\right)^2$, or 36, to both sides to complete the square.*

$$(x-6)^2 = 16y$$ *Factor and simplify.*

$$\frac{1}{16}(x-6)^2 = y$$ *Divide both sides by 16.*

$$y = \frac{1}{16}(x-6)^2$$ *Rewrite in standard form.*

Because the conic is of the form $y - k = \frac{1}{4p}(x-h)^2$, it is a parabola with vertex $(6, 0)$ and $p = 4$, and it opens upward. The focus is $(6, 4)$ and the directrix is $y = -4$.




**B** $x^2 + 4y^2 + 4x - 24y + 36 = 0$

$$x^2 + 4x + \square + 4y^2 - 24y + \square = -36 + \square + \square$$ *Rearrange to prepare for completing the square in x and y.*

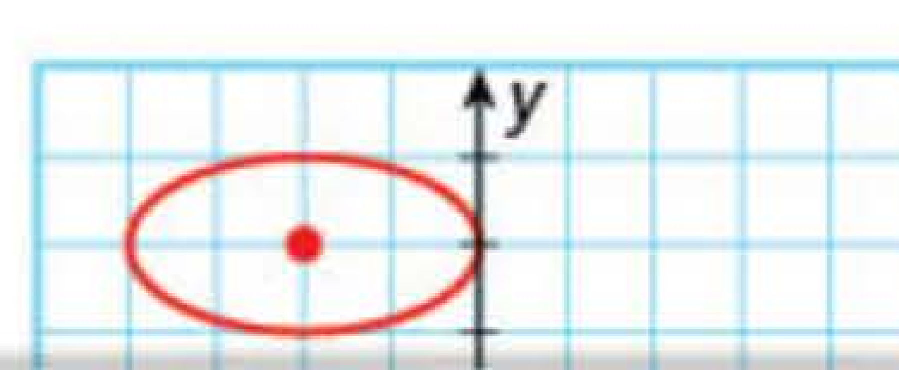
$$x^2 + 4x + \square + 4(y^2 - 6y + \square) = -36 + \square + \square$$ *Factor 4 from the y terms.*

$$x^2 + 4x + \left(\frac{4}{2}\right)^2 + 4\left[y^2 - 6y + \left(-\frac{6}{2}\right)^2\right] = -36 + \left(\frac{4}{2}\right)^2 + 4\left(-\frac{6}{2}\right)^2$$ *Complete both squares.*

$$(x+2)^2 + 4(y-3)^2 = 4$$ *Factor and simplify.*

$$\frac{(x+2)^2}{4} + \frac{(y-3)^2}{1} = 1$$ *Divide both sides by 4.*

Because the conic is of the form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, it is an ellipse with



**B** $x^2 + 4y^2 + 4x - 24y + 36 = 0$

$$x^2 + 4x + \square + 4y^2 - 24y + \square = -36 + \square + \square$$ *Rearrange to prepare for completing the square in x and y.*

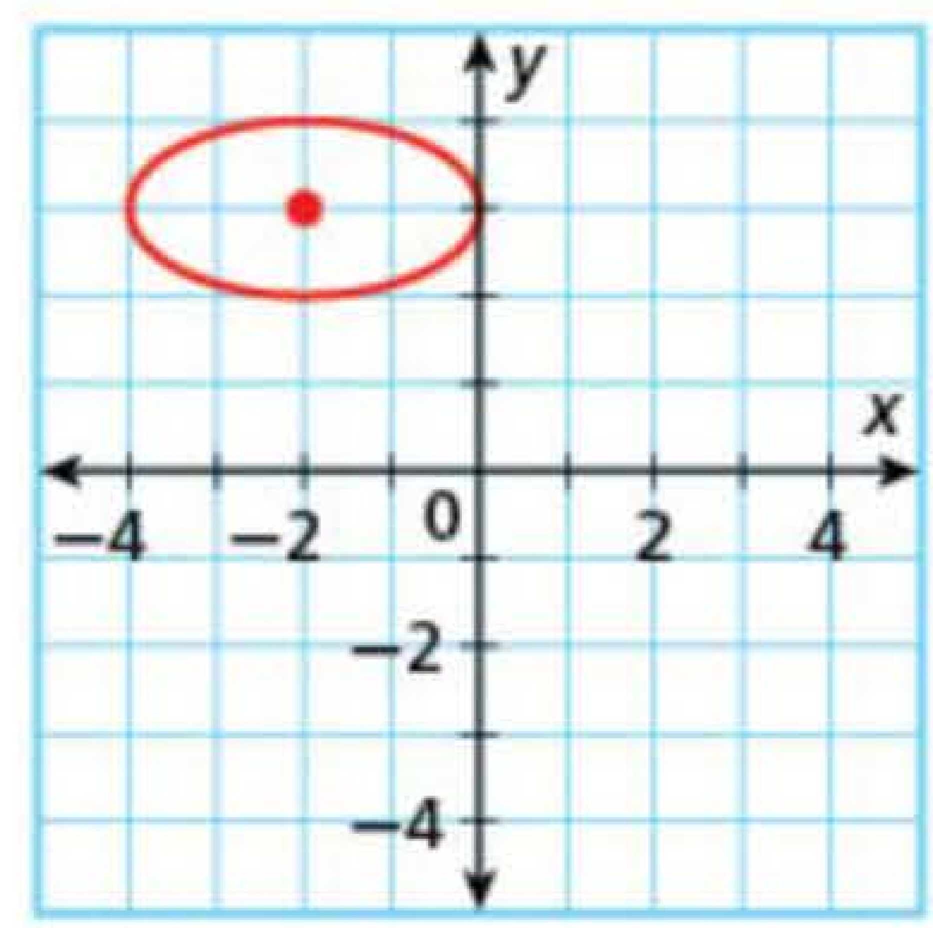
$$x^2 + 4x + \square + 4(y^2 - 6y + \square) = -36 + \square + \square$$ *Factor 4 from the y terms.*

$$x^2 + 4x + \left(\frac{4}{2}\right)^2 + 4\left[y^2 - 6y + \left(-\frac{6}{2}\right)^2\right] = -36 + \left(\frac{4}{2}\right)^2 + 4\left(-\frac{6}{2}\right)^2$$ *Complete both squares.*

$$(x + 2)^2 + 4(y - 3)^2 = 4$$ *Factor and simplify.*

$$\frac{(x + 2)^2}{4} + \frac{(y - 3)^2}{1} = 1$$ *Divide both sides by 4.*

Because the conic is of the form $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$, it is an ellipse with center $(-2, 3)$, horizontal major axis length 4, and minor axis length 2. The co-vertices are $(-2, 4)$ and $(-2, 2)$, and the vertices are $(-4, 3)$ and $(0, 3)$.




Tap the button to view Example 3 in StepReveal.

View in Motion

Tap the button to watch the lesson tutorial video for extra instruction.

Play Video

## Check It Out!

**Find the standard form of each equation by completing the square. Then identify and graph each conic.**

**3a.** $y^2 - 9x + 16y + 64 = 0$

**3b.** $16x^2 + 9y^2 - 128x + 108y + 436 = 0$

EXAMPLE 4

## Aviation Application

**At an air show, an airplane makes a dive that can be modeled by the equation $-4x^2 + 16y^2 - 16x + 32y - 64 = 0$, measured in hundreds of feet, with the ground represented by the $x$-axis. How close to the ground does the airplane pass?**

The graph of $-4x^2 + 16y^2 - 16x + 32y - 64 = 0$ is a conic section. Write the equation in standard form.

$$-4x^2 - 16x + \square + 16y^2 + 32y + \square = 64 + \square + \square$$

*Rearrange to prepare for completing the square in x and y.*

$$-4(x^2 + 4x + \square) + 16(y^2 + 2y + \square) = 64 + \square + \square$$

*Factor –4 from the x terms and 16 from the y terms.*

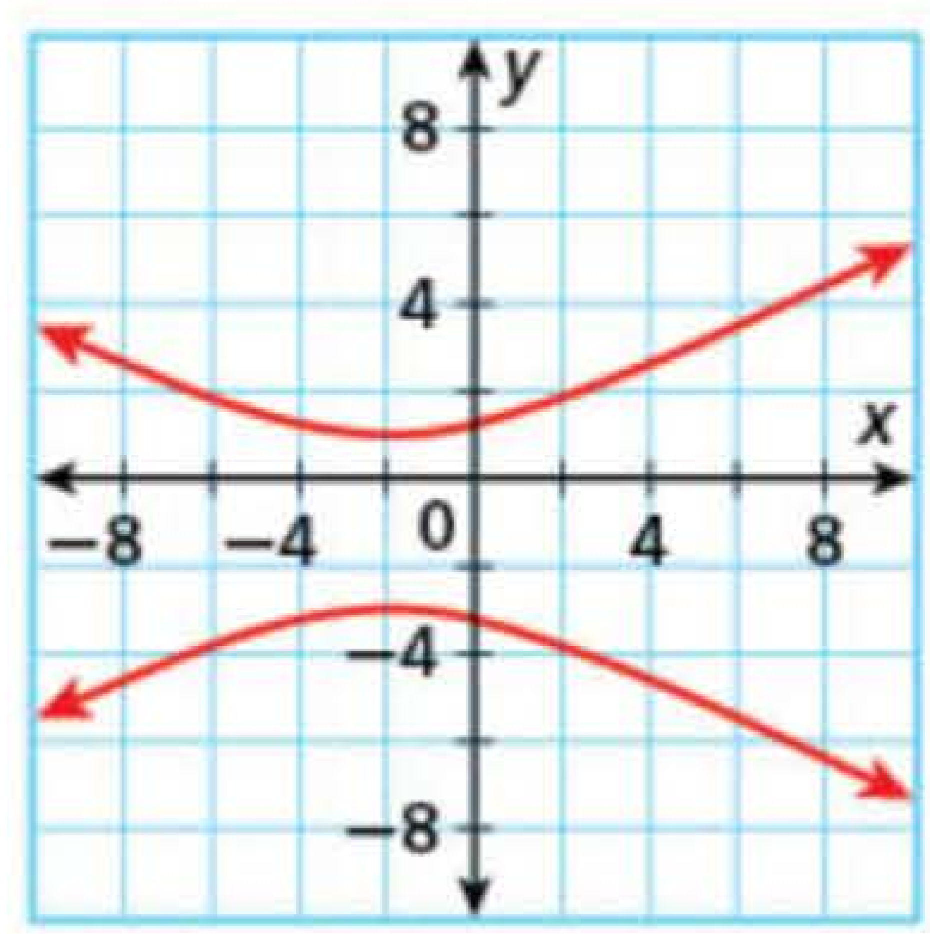
$$-4\left[x^2 + 4x + \left(\frac{4}{2}\right)^2\right] + 16\left[y^2 + 2y + \left(\frac{2}{2}\right)^2\right] = 64 - 4\left(\frac{4}{2}\right)^2 + 16\left(\frac{2}{2}\right)^2$$

*Complete both squares.*

$$16(y + 1)^2 - 4(x + 2)^2 = 64 \qquad \textit{Simplify.}$$

$$\frac{(y + 1)^2}{4} - \frac{(x + 2)^2}{16} = 1 \qquad \textit{Divide both sides by 64.}$$

Because the conic is of the form $\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$, it is a hyperbola with vertical transverse axis length 4 and center $(-2, -1)$. The vertices are then $(-2, 1)$ and $(-2, -3)$. Because distance above ground is always positive, the airplane will be on the upper branch of the hyperbola. The relevant vertex is $(-2, 1)$ with $y$-coordinate 1.



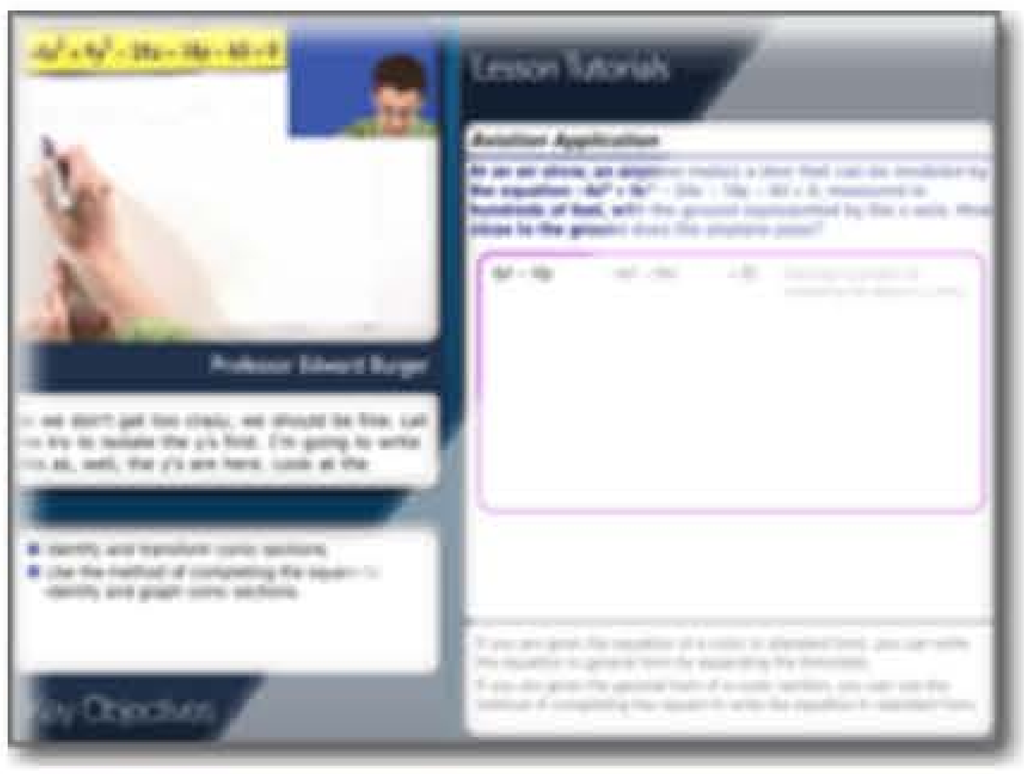

The minimum height of the plane is 100 feet.

Tap the button to view Example 4 in StepReveal.

View in Motion

Tap the button to view Example 4 in StepReveal.

View in Motion



Tap the button to watch the lesson tutorial video for extra instruction.

Play Video

## Check It Out!

**4.** An airplane makes a dive that can be modeled by the equation $-16x^2 + 9y^2 + 96x + 36y - 252 = 0$, measured in hundreds of feet. How close to the ground does the airplane pass?

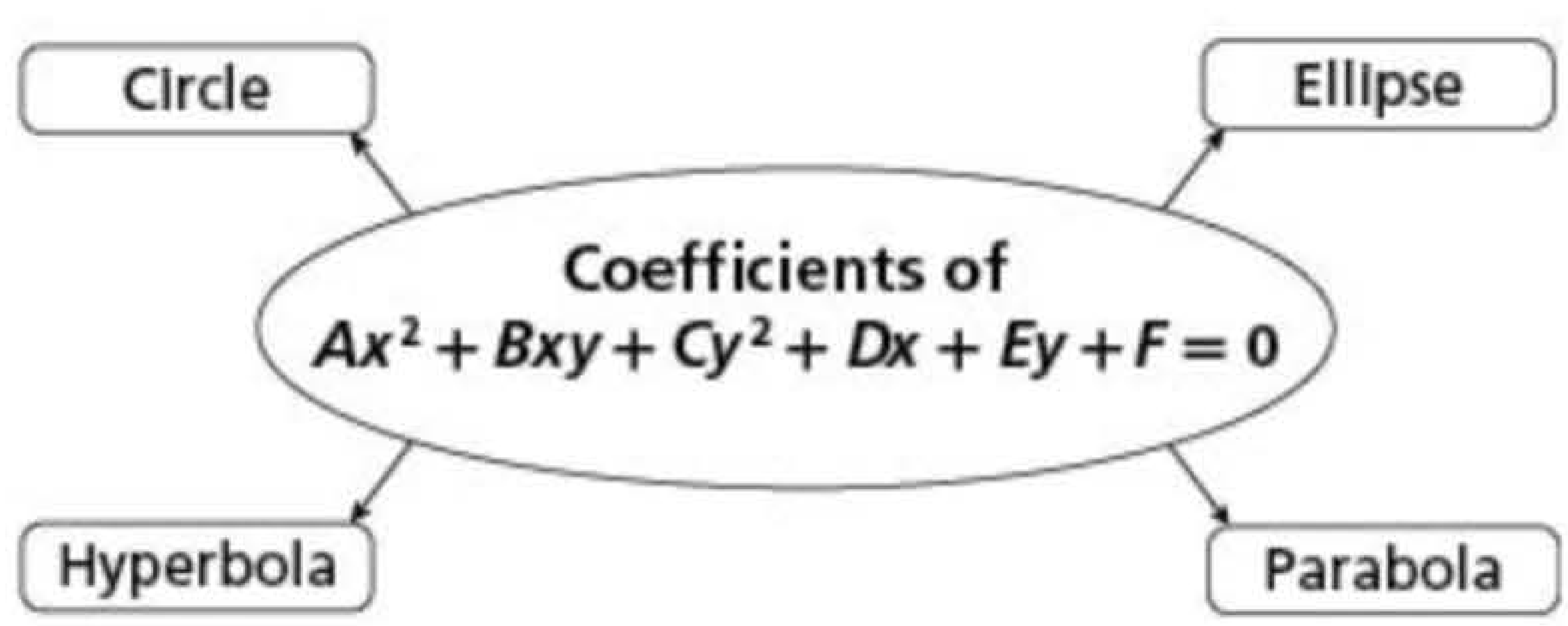
Try These Problems

## THINK AND DISCUSS

**1.** In the equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, if $B = 0$, what must be true about either $A$ or $C$ for the equation to represent a parabola?

**2.** When solving by completing the square, what must be added to both sides of the equation if one side has $5x^2 - 30x$? Explain.

**3. GET ORGANIZED** Copy and complete the graphic organizer. Give an example of coefficients for each conic section in general form.




Know it! Note

# GUIDED PRACTICE



**Identify the conic section that each equation represents.**

see example 1

1. $\dfrac{(x+4)^2}{2^2}+\dfrac{(y-3)^2}{3^2}=1$

2. $\dfrac{(x-8)^2}{5^2}-\dfrac{y^2}{5^2}=1$

3. $y+9=4(x-1)^2$

4. $(x-2)^2+(y-6)^2=13^2$

see example 2

5. $12x^2+18y^2-8x+9y-10=0$

6. $-4y^2+15x+12y-8=0$

7. $10x^2+15xy+10y^2+15x+25y+9=0$

8. $6x^2=14x+12y^2-16y+20$

**Find the standard form of each equation by completing the square. Then identify and graph each conic.**

see example 3

9. $x^2+y^2-16x+10y+53=0$

10. $x^2+14x-12y+97=0$

$5^2$ $5^2$

3 $y + 9 = 4(x - 1)^2$

4 $(x - 2)^2 + (y - 6)^2 = 13^2$

see example 2

5 $12x^2 + 18y^2 - 8x + 9y - 10 = 0$

6 $-4y^2 + 15x + 12y - 8 = 0$

7 $10x^2 + 15xy + 10y^2 + 15x + 25y + 9 = 0$

8 $6x^2 = 14x + 12y^2 - 16y + 20$

**Find the standard form of each equation by completing the square. Then identify and graph each conic.**

see example 3

9 $x^2 + y^2 - 16x + 10y + 53 = 0$

10 $x^2 + 14x - 12y + 97 = 0$

11 $25x^2 + 9y^2 + 72y - 81 = 0$

12 $16x^2 + 36y^2 + 160x - 432y + 1120 = 0$

see example 4

13

**Multi-Step** A moth is circling an outdoor light in a path that can be modeled by the equation $4x^2 + 9y^2 - 108y = -288$, measured in inches. How close does the moth pass to a lizard located at the origin?

Homework Help

#35

#37

#39

#41

#43

**Identify the conic section that each equation represents.**

14. $\frac{(y-11)^2}{2^2} - \frac{(x+15)^2}{9^2} = 1$

15. $x - 4 = \frac{1}{16}(y-3)^2$

16. $(x+2)^2 + (y-4)^2 = 3^2$

17. $\frac{(x+2)^2}{6^2} + \frac{(y-7)^2}{8^2} = 1$

18. $12x^2 - 18y^2 - 18x - 12y + 12 = 0$

19. $7x^2 + 28x - 29y - 16 = 0$

20. $-12x^2 - 3y^2 + 7x + 9y - 5 = 0$

21. $12x^2 + 9y^2 - 2xy + 9 = 8y - 3y^2$

**Find the standard form of each equation by completing the square. Then identify and graph each conic.**

22. $x^2 + 20x - 4y + 100 = 0$

23. $x^2 + y^2 - 8y - 33 = 0$

24. $9x^2 + 36y^2 - 72x - 180 = 0$

25. $25x^2 - 4y^2 - 72y - 424 = 0$

26. $x^2 - 2x - 20y - 79 = 0$

27. $x^2 + y^2 + 10x + 4y + 9 = 0$

28. $64x^2 + 49y^2 + 256x - 196y - 2684 = 0$

29. $9x^2 - 4y^2 + 18x + 56y - 223 = 0$

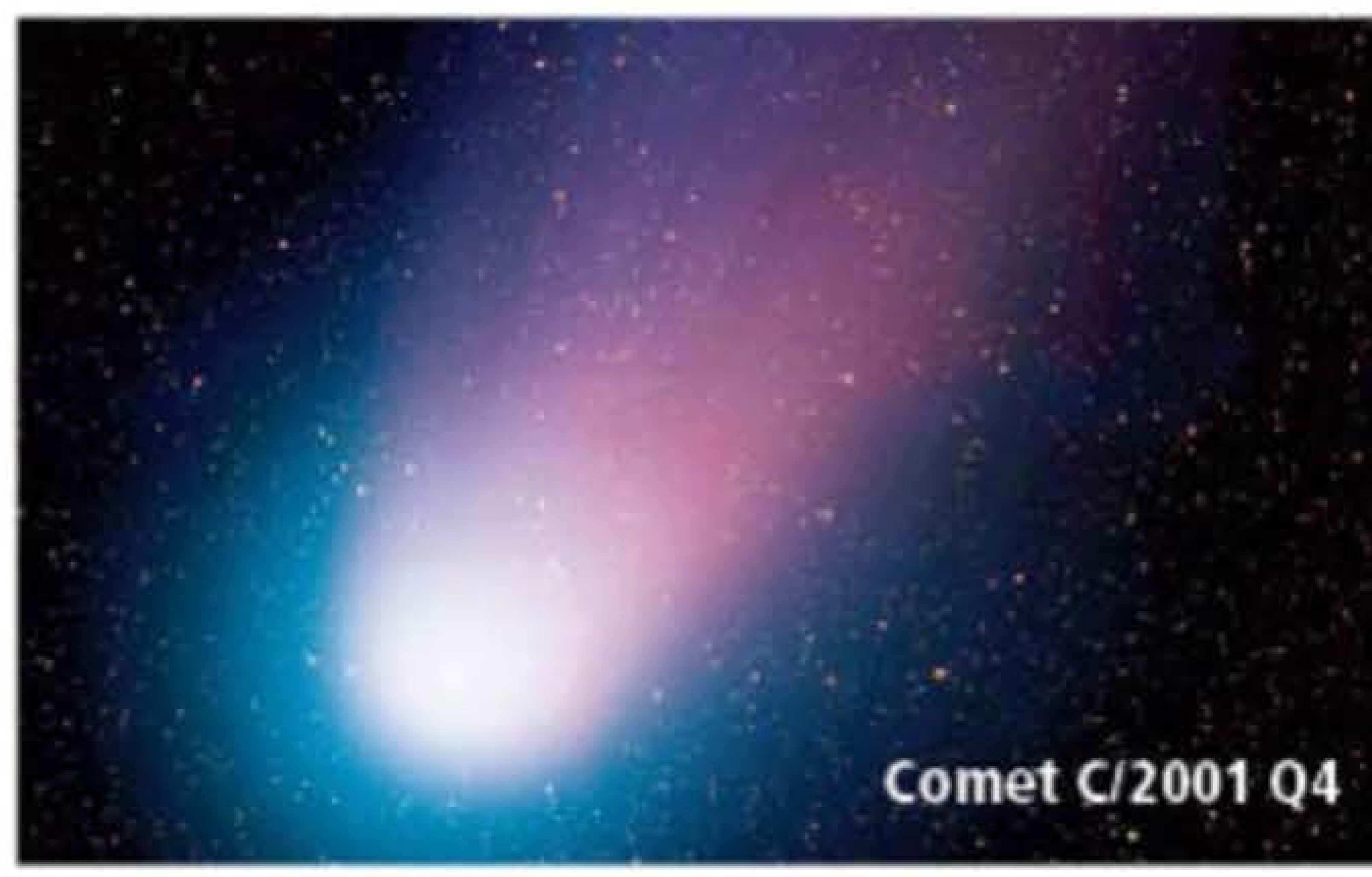
30. $y^2 + 6x + 12y - 6 = 0$

31. $x^2 + y^2 - 5x + 9y + 10.5 = 0$

32. **Astronomy** Scientists find that the path of a comet as it travels around the Sun can be modeled by the function $225x^2 + 64y^2 + 7650x + 50{,}625 = 0$, with the Sun as one focus.

a. Write the equation in standard form.

b. If measurements are in millions of miles, about how close will the comet come to the sun?



Comet C/2001 Q4



MULTI-STEP TEST PREP

33. A water-skier is towed along a path that can be modeled by $25x^2 + 4y^2 + 300x - 24y + 836 = 0$. Each unit of the coordinate plane represents 10 m.

a. What is the shape of the water-skier's path?

b. The edge of a dock is represented by the $y$-axis. How close does the water-skier come to the dock?

c. A second water-skier is towed along the same path. What is the maximum possible distance between the two water-skiers?

Link

**Write each equation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.**

34. $(x-7)^2 + (y+12)^2 = 81$

35. $\frac{(x-5)^2}{25} + \frac{(y+8)^2}{36} = 1$

36. $\frac{(x+10)^2}{49} - \frac{(y-6)^2}{81} = 1$

**Determine whether the origin lies inside, outside, or on the graph of each equation.**

37. $36x^2 + 4y^2 - 432x + 1152 = 0$

38. $4x^2 + 36y^2 - 48x = 0$

39. $16x^2 + 64y^2 - 192x + 16y - 447 = 0$

40. $3x^2 + 3y^2 = 147$

41. **Multi-Step** A model of the solar system includes a satellite orbiting the Moon on a path that can be modeled by the equation $6x^2 + 6y^2 = 24$, measured in centimeters (1 cm:10,000 km). If the Moon is located at the point $(0, 38.4)$, how close will the satellite pass to the Moon in the model?

42. **Critical Thinking** What does the graph of $x^2 - xy = 0$ look like? Explain.

43. **Agriculture** A farmer is planning to fence in part of the farm. Placing the farmhouse at the origin, the farmer finds that the path for the fence can be modeled by the equation $x^2 + y^2 - 80x - 60y - 37{,}500 = 0$, measured in feet.
   a. Write the equation in standard form.
   b. Find the area enclosed by the fence.
   c. Is the farmhouse inside or outside of the fence?

44. **/// ERROR ANALYSIS ///** In which case below was the conic section $4y^2 + 3x - 12y = 2x^2 + 18$ identified incorrectly? Explain the error.

**A**

$4y^2 + 3x - 12y = 2x^2 + 18$
$-2x^2 + 4y^2 + 3x - 12y - 18 = 0$
$A = -2, B = 0, C = 4$
$B^2 - 4AC = 0 - 4(-2)(4)$
$B^2 - 4AC = 32$
The equation represents a hyperbola.

**B**

$4y^2 + 3x - 12y = 2x^2 + 18$
$A = 2, B = 0, C = 4$
$B^2 - 4AC = 0 - 4(2)(4)$
$B^2 - 4AC = 32$
$A \neq C$
The equation represents an ellipse.

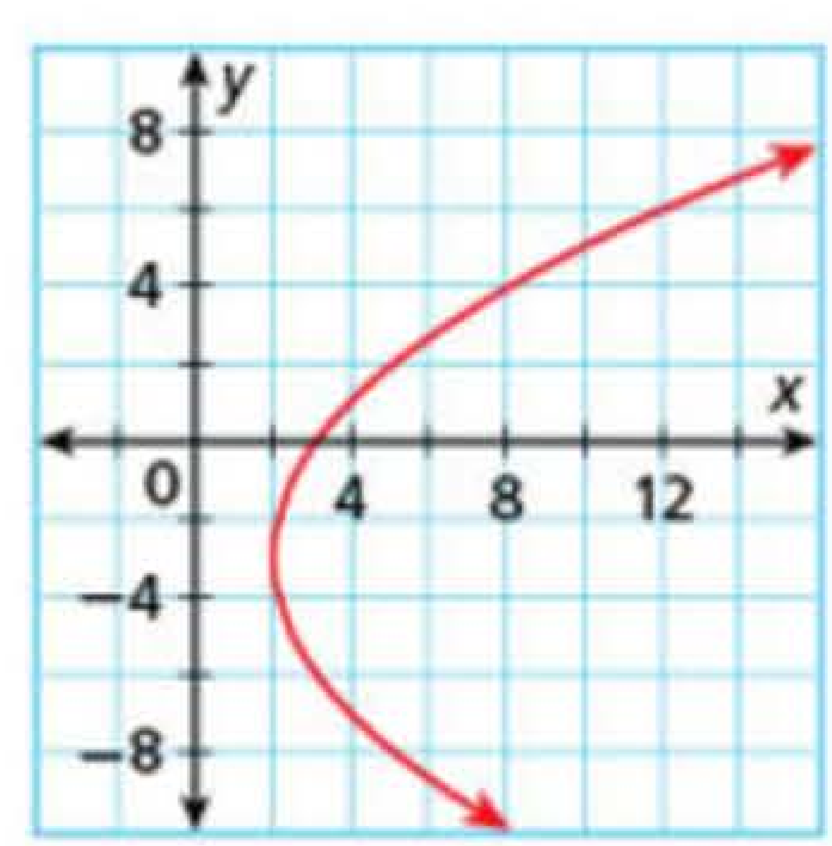
45. **Sports** The path followed by a baseball after it is hit can be modeled by the equation $2x^2 - 800x + 1000y - 4000 = 0$, measured in feet.
   a. Write the equation in standard form.
   b. What is the maximum height of the ball?
   c. What was the height of the ball when it was hit?
   d. **What if...?** How would changing the 4000 in the equation to 5000 change your answers to parts **b** and **c**?

46. **Write About It** Compare the equations and graphs of parabolas and hyperbolas.



## Test Prep

47. Which of the following is the equation for the graph shown?
   (A) $3y^2 - 24x + 18y + 75 = 0$
   (B) $5x^2 + 30x - 40y + 125 = 0$
   (C) $2x^2 - 3y^2 + 18x - 24y + 75 = 0$
   (D) $3x^2 + 2y^2 - 24x + 18y + 125 = 0$




48. The graph of $9x^2 + 15x - 9y^2 - 15y + 25 = 0$ is which of the following?
   (F) Circle
   (G) Ellipse
   (H) Hyperbola
   (J) Parabola

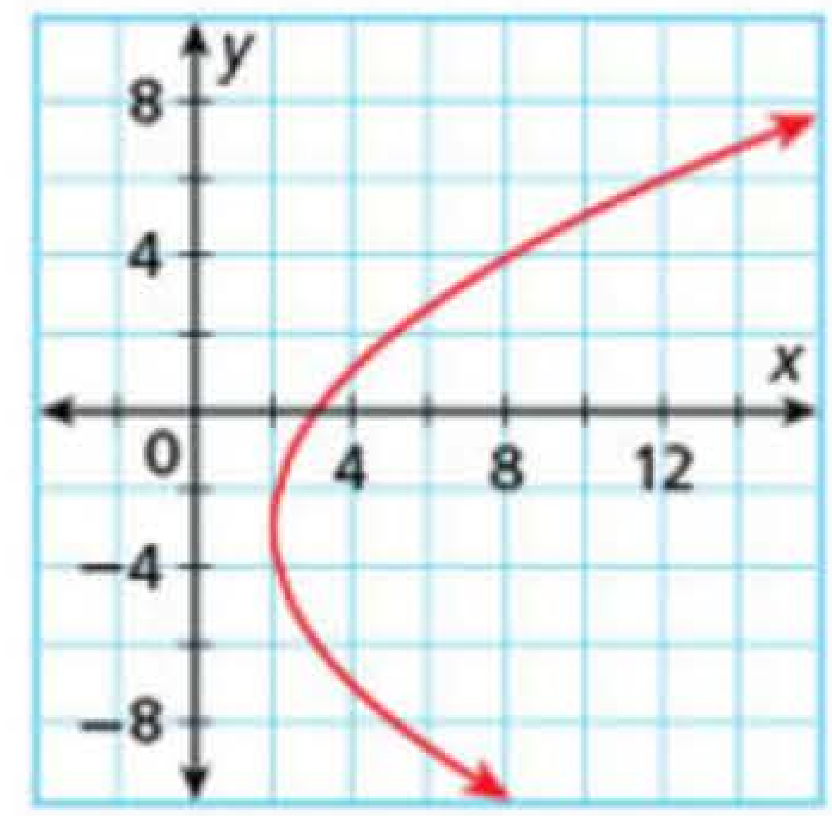
49. Which of the following is the equation for the graph shown?

46. **Write About It** Compare the equations and graphs of parabolas and hyperbolas.



## Test Prep

47. Which of the following is the equation for the graph shown?

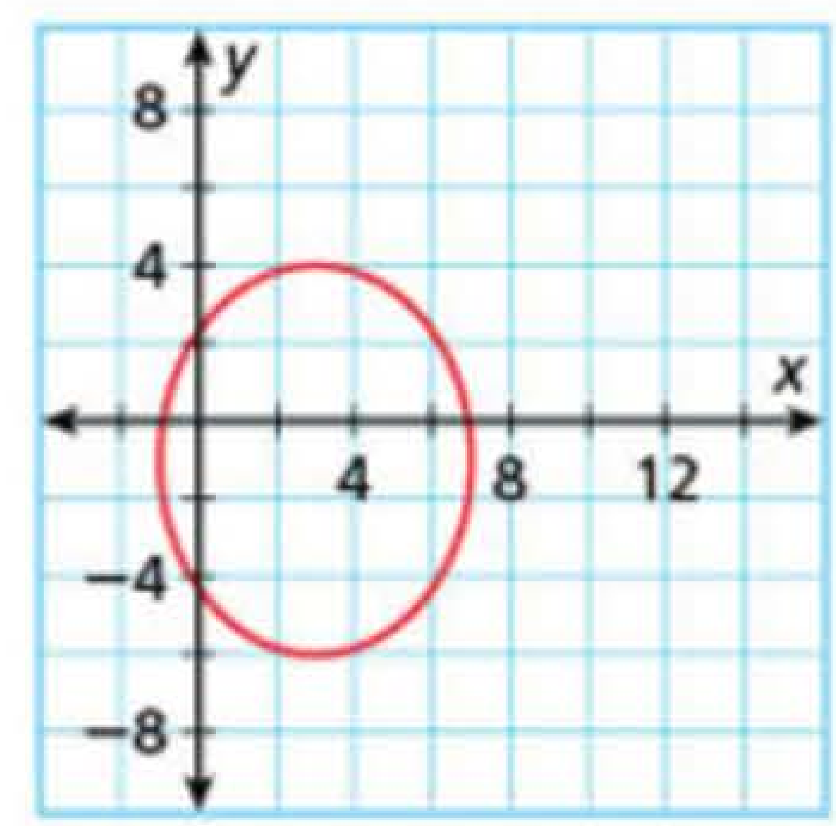



Ⓐ $3y^2 - 24x + 18y + 75 = 0$
Ⓑ $5x^2 + 30x - 40y + 125 = 0$
Ⓒ $2x^2 - 3y^2 + 18x - 24y + 75 = 0$
Ⓓ $3x^2 + 2y^2 - 24x + 18y + 125 = 0$

48. The graph of $9x^2 + 15x - 9y^2 - 15y + 25 = 0$ is which of the following?

Ⓕ Circle　Ⓖ Ellipse
Ⓗ Hyperbola　Ⓙ Parabola

49. Which of the following is the equation for the graph shown?




Ⓐ $25x^2 + 25y^2 - 150x + 32y - 159 = 0$
Ⓑ $25x^2 - 150x + 32y = 159$
Ⓒ $25x^2 - 150x = 16y^2 - 32y + 159$
Ⓓ $16y^2 + 32y - 159 = 150x - 25x^2$

50. **Short Response** Write the equation $x^2 + y^2 + 8x - 6y + 16 = 0$ in standard form, and identify the conic section that it represents. What are the coordinates of the center?

Check Answers

## CHALLENGE AND EXTEND

In order to graph the general form of conic sections, $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, use the quadratic formula,

$$y = \frac{-(Bx + E) \pm \sqrt{(Bx + E)^2 - 4C(Ax^2 + Dx + F)}}{2C},$$ and a graphing calculator.

51. Graph $4x^2 + 8xy - 9y^2 - 36 = 0$.

52. Graph $9x^2 - 12xy + 16y^2 - 144 = 0$.

53. What effect does the term $Bxy$ have on the graph?

54. **What if...?** What happens to the formula if $C = 0$?

Check Answers