COMMON CORE

10-4 Inverses of Trigonometric Functions

CC.9-12.F.TF.6 (+) Understand that restricting a trigonometric function…allows its inverse to be constructed. *Also* **CC.9-12.F.TF.7* (+)**

Objectives

Evaluate inverse trigonometric functions.

Use trigonometric equations and inverse trigonometric functions to solve problems.

Vocabulary

inverse sine function
inverse cosine function
inverse tangent function

Who uses this?

Hikers can use inverse trigonometric functions to navigate in the wilderness. (See Example 3.)

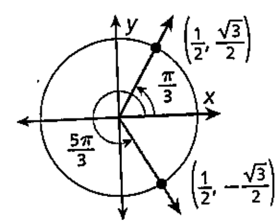
You have evaluated trigonometric functions for a given angle. You can also find the measure of angles given the value of a trigonometric function by using an *inverse trigonometric* relation.

Function	Inverse Relation
$\sin\theta = a$	$\sin^{-1} a = \theta$
$\cos\theta = a$	$\cos^{-1} a = \theta$
$\tan\theta = a$	$\tan^{-1} a = \theta$

Reading Math

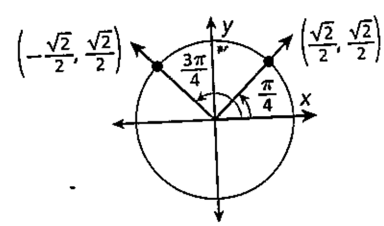
The expression $\sin^{-1}$ is read as "the inverse sine." In this notation, $^{-1}$ indicates the *inverse* of the sine function, NOT the *reciprocal* of the sine function.

The inverses of the trigonometric functions are not functions themselves because there are many values of θ for a particular value of a. For example, suppose that you want to find $\cos^{-1}\frac{1}{2}$. Based on the unit circle, angles that measure $\frac{\pi}{3}$ and $\frac{5\pi}{3}$ radians have a cosine of $\frac{1}{2}$. So do all angles that are coterminal with these angles.



EXAMPLE 1 **Finding Trigonometric Inverses**

Find all possible values of $\sin^{-1}\frac{\sqrt{2}}{2}$.



Step 1 Find the values between 0 and 2π radians for which $\sin\theta$ is equal to $\frac{\sqrt{2}}{2}$.

$$\frac{\sqrt{2}}{2} = \sin\frac{\pi}{4}, \qquad \frac{\sqrt{2}}{2} = \sin\frac{3\pi}{4}$$

Use y-coordinates of points on the unit circle.

Step 2 Find the angles that are coterminal with angles measuring $\frac{\pi}{4}$ and $\frac{3\pi}{4}$ radians.

$$\frac{\pi}{4} + (2\pi)n, \qquad \frac{3\pi}{4} + (2\pi)n$$

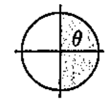
Add integer multiples of 2π radians, where n is an integer.

CHECK IT OUT! 1. Find all possible values of $\tan^{-1}1$.

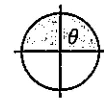
Because more than one value of θ produces the same output value for a given trigonometric function, it is necessary to restrict the domain of each trigonometric function in order to define the inverse trigonometric functions.

Trigonometric functions with restricted domains are indicated with a capital letter. The domains of the Sine, Cosine, and Tangent functions are restricted as follows.

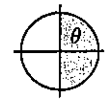
$\text{Sin}\,\theta = \sin\theta$ for $\left\{\theta \mid -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}\right\}$ *θ is restricted to Quadrants I and IV.*



$\text{Cos}\,\theta = \cos\theta$ for $\left\{\theta \mid 0 \le \theta \le \pi\right\}$ *θ is restricted to Quadrants I and II.*



$\text{Tan}\,\theta = \tan\theta$ for $\left\{\theta \mid -\frac{\pi}{2} < \theta < \frac{\pi}{2}\right\}$ *θ is restricted to Quadrants I and IV.*



These functions can be used to define the inverse trigonometric functions. For each value of a in the domain of the inverse trigonometric functions, there is only one value of θ. Therefore, even though $\tan^{-1} 1$ has many values, $\text{Tan}^{-1} 1$ has only one value.

Know it! Note

Inverse Trigonometric Functions

WORDS	SYMBOL	DOMAIN	RANGE
The **inverse sine function** is $\text{Sin}^{-1} a = \theta$, where $\text{Sin}\,\theta = a$.	$\text{Sin}^{-1} a$	$\{a \mid -1 \le a \le 1\}$	$\left\{\theta \mid -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}\right\}$ $\{\theta \mid -90° \le \theta \le 90°\}$
The **inverse cosine function** is $\text{Cos}^{-1} a = \theta$, where $\text{Cos}\,\theta = a$.	$\text{Cos}^{-1} a$	$\{a \mid -1 \le a \le 1\}$	$\{\theta \mid 0 \le \theta \le \pi\}$ $\{\theta \mid 0° \le \theta \le 180°\}$
The **inverse tangent function** is $\text{Tan}^{-1} a = \theta$, where $\text{Tan}\,\theta = a$.	$\text{Tan}^{-1} a$	$\{a \mid -\infty < a < \infty\}$	$\left\{\theta \mid -\frac{\pi}{2} < \theta < \frac{\pi}{2}\right\}$ $\{\theta \mid -90° < \theta < 90°\}$

Reading Math

The inverse trigonometric functions are also called the arcsine, arccosine, and arctangent functions.

EXAMPLE 2 Evaluating Inverse Trigonometric Functions

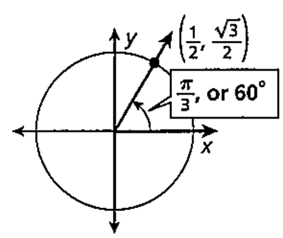
Evaluate each inverse trigonometric function. Give your answer in both radians and degrees.

A $\text{Cos}^{-1}\frac{1}{2}$

$\frac{1}{2} = \text{Cos}\,\theta$ *Find the value of θ for $0 \le \theta \le \pi$ whose Cosine is $\frac{1}{2}$.*

$\frac{1}{2} = \text{Cos}\,\frac{\pi}{3}$ *Use x-coordinates of points on the unit circle.*

$\text{Cos}^{-1}\frac{1}{2} = \frac{\pi}{3}$, or $\text{Cos}^{-1}\frac{1}{2} = 60°$



B $\text{Sin}^{-1} 2$

The domain of the inverse sine function is $\{a \mid -1 \le a \le 1\}$. Because 2 is outside this domain, $\text{Sin}^{-1} 2$ is undefined.

CHECK IT OUT! **Evaluate each inverse trigonometric function. Give your answer in both radians and degrees.**

2a. $\text{Sin}^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

2b. $\text{Cos}^{-1} 0$

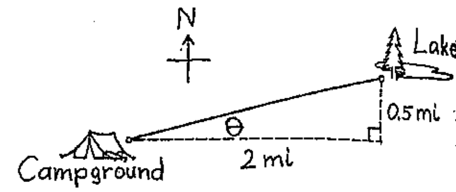
You can solve trigonometric equations by using trigonometric inverses.

EXAMPLE 3 *Navigation Application*

A group of hikers plans to walk from a campground to a lake. The lake is 2 miles east and 0.5 mile north of the campground. To the nearest degree, in what direction should the hikers head?

Step 1 Draw a diagram.

The hikers' direction should be based on θ, the measure of an acute angle of a right triangle.



Caution!

If the answer on your calculator screen is 0.2449786631 when you enter $\tan^{-1}(0.25)$, your calculator is set to radian mode instead of degree mode.

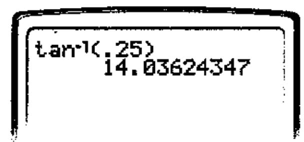
Step 2 Find the value of θ.

$\tan\theta = \dfrac{\text{opp.}}{\text{adj.}}$ — *Use the tangent ratio.*

$\tan\theta = \dfrac{0.5}{2} = 0.25$ — *Substitute 0.5 for opp. and 2 for adj. Then simplify.*

$\theta = \text{Tan}^{-1}0.25$

$\theta \approx 14°$



The hikers should head 14° north of east.

CHECK IT OUT! **Use the information given above to answer the following.**

3. An unusual rock formation is 1 mile east and 0.75 mile north of the lake. To the nearest degree, in what direction should the hikers head from the lake to reach the rock formation?

EXAMPLE 4 Solving Trigonometric Equations

Solve each equation to the nearest tenth. Use the given restrictions.

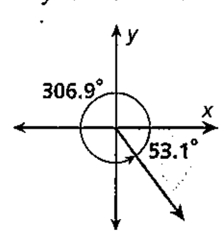
A $\cos\theta = 0.6$, **for** $0° \le \theta \le 180°$

The restrictions on θ are the same as those for the inverse cosine function.

$\theta = \text{Cos}^{-1}(0.6) \approx 53.1°$ — *Use the inverse cosine function on your calculator.*

B $\cos\theta = 0.6$, **for** $270° < \theta < 360°$

The terminal side of θ is restricted to Quadrant IV. Find the angle in Quadrant IV that has the same cosine value as 53.1°.



$\theta \approx 360° - 53.1° \approx 306.9°$ — *θ has a reference angle of 53.1°, and $270° < \theta < 360°$.*

CHECK IT OUT! **Solve each equation to the nearest tenth. Use the given restrictions.**

4a. $\tan\theta = -2$, for $-90° < \theta < 90°$

4b. $\tan\theta = -2$, for $90° < \theta < 180°$

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Given that θ is an acute angle in a right triangle, describe the measurements that you need to know to find the value of θ by using the inverse cosine function.
2. Explain the difference between $\tan^{-1} a$ and $\text{Tan}^{-1} a$.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, give the indicated property of the inverse trigonometric functions.

Know it! Note

Symbols	Domains
Inverse Trigonometric Functions	
Associated quadrants	Ranges

10-4 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

1. **Vocabulary** Explain how the inverse tangent function differs from the reciprocal of the tangent function.

SEE EXAMPLE 1

Find all possible values of each expression.

2. $\sin^{-1}\left(-\frac{1}{2}\right)$
3. $\tan^{-1} \frac{\sqrt{3}}{3}$
4. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

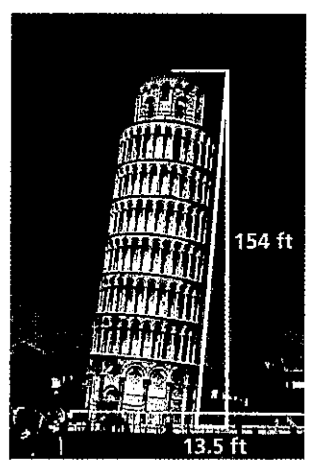
SEE EXAMPLE 2

Evaluate each inverse trigonometric function. Give your answer in both radians and degrees.

5. $\text{Cos}^{-1} \frac{\sqrt{3}}{2}$
6. $\text{Tan}^{-1} 1$
7. $\text{Cos}^{-1} 2$
8. $\text{Tan}^{-1}\left(-\sqrt{3}\right)$
9. $\text{Sin}^{-1} \frac{\sqrt{2}}{2}$
10. $\text{Sin}^{-1} 0$

SEE EXAMPLE 3

11. **Architecture** A point on the top of the Leaning Tower of Pisa is shifted about 13.5 ft horizontally compared with the tower's base. To the nearest degree, how many degrees does the tower tilt from vertical?



SEE EXAMPLE 4

Solve each equation to the nearest tenth. Use the given restrictions.

12. $\tan\theta = 1.4$, for $-90° < \theta < 90°$
13. $\tan\theta = 1.4$, for $180° < \theta < 270°$
14. $\cos\theta = -0.25$, for $0 \le \theta \le 180°$
15. $\cos\theta = -0.25$, for $180° < \theta < 270°$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
16–18	1
19–24	2
25	3
26–29	4

Extra Practice

See Extra Practice for more Skills Practice and Applications Practice exercises.

Find all possible values of each expression.

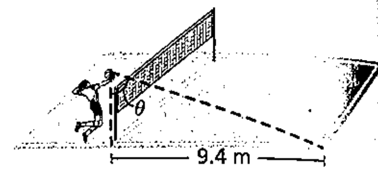
16. $\cos^{-1} 1$ **17.** $\sin^{-1} \frac{\sqrt{3}}{2}$ **18.** $\tan^{-1}(-1)$

Evaluate each inverse trigonometric function. Give your answer in both radians and degrees.

19. $\text{Sin}^{-1} \frac{\sqrt{3}}{2}$ **20.** $\text{Cos}^{-1}(-1)$ **21.** $\text{Tan}^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

22. $\text{Cos}^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ **23.** $\text{Tan}^{-1}\sqrt{3}$ **24.** $\text{Sin}^{-1}\sqrt{3}$

25. Volleyball A volleyball player spikes the ball from a height of 2.44 m. Assume that the path of the ball is a straight line. To the nearest degree, what is the maximum angle θ at which the ball can be hit and land within the court?



LINK

Aviation

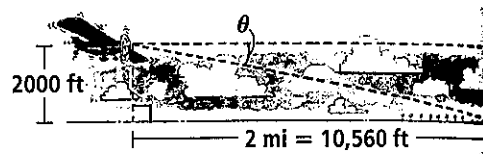
A flight simulator is a device used in training pilots that mimics flight conditions as realistically as possible. Some flight simulators involve full-size cockpits equipped with sound, visual, and motion systems.

Solve each equation to the nearest tenth. Use the given restrictions.

26. $\sin\theta = -0.75$, for $-90° \le \theta \le 90°$ **27.** $\sin\theta = -0.75$, for $180° < \theta < 270°$

28. $\cos\theta = 0.1$, for $0° \le \theta \le 180°$ **29.** $\cos\theta = 0.1$, for $270° < \theta < 360°$

30. Aviation The pilot of a small plane is flying at an altitude of 2000 ft. The pilot plans to start the final descent toward a runway when the horizontal distance between the plane and the runway is 2 mi. To the nearest degree, what will be the angle of depression θ from the plane to the runway at this point?



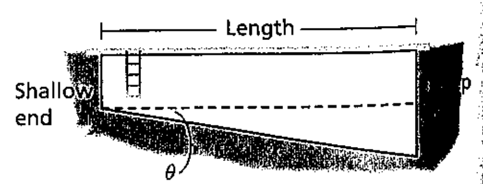
31. Multi-Step The table shows the dimensions of three pool styles offered by a construction company.

Pool Style	Length (ft)	Shallow End Depth (ft)	Deep En Depth (f
A	38	3	8
B	25	2	6
C	50	2.5	7

a. To the nearest tenth of a degree, what angle θ does the bottom of each pool make with the horizontal?

b. Which pool style's bottom has the steepest slope? Explain.

c. What if...? If the slope of the bottom of a pool can be no greater than $\frac{1}{6}$, what is the greatest angle θ that the bottom of the pool can make with the horizontal? Round to the nearest tenth of a degree.



32. Navigation Lines of longitude are closer together near the poles than at the equator. The formula for the length ℓ of 1° of longitude in miles is $\ell = 69.0933 \cos\theta$, where θ is the latitude in degrees.

a. At what latitude, to the nearest degree, is the length of a degree of longitude approximately 59.8 miles?

b. To the nearest mile, how much longer is the length of a degree of longitude at the equator, which has a latitude of 0°, than at the Arctic Circle, which has a latitude of about 66°N?

MULTI-STEP TEST PREP

33. Giant kelp is a seaweed that typically grows about 100 ft in height, but may reach as high as 175 ft.

   a. A diver positions herself 10 ft from the base of a giant kelp so that her eye level is 5 ft above the ocean floor. If the kelp is 100 ft in height, what would be the angle of elevation from the diver to the top of the kelp? Round to the nearest tenth of a degree.

   b. The angle of elevation from the diver's eye level to the top of a giant kelp whose base is 30 ft away is 75.5°. To the nearest foot, what is the height of the kelp?

Find each value.

34. $\text{Cos}^{-1}(\cos 0.4)$ **35.** $\tan(\text{Tan}^{-1} 0.7)$ **36.** $\sin(\text{Cos}^{-1} 0)$

37. Critical Thinking Explain why the domain of the Cosine function is different from the domain of the Sine function.

38. Write About It Is the statement $\text{Sin}^{-1}(\sin\theta) = \theta$ true for all values of θ? Explain.

TEST PREP

39. For which equation is the value of θ in radians a positive value?

Ⓐ $\text{Cos}\,\theta = -\frac{1}{2}$ Ⓑ $\text{Tan}\,\theta = -\frac{\sqrt{3}}{3}$ Ⓒ $\text{Sin}\,\theta = -\frac{\sqrt{3}}{2}$ Ⓓ $\text{Sin}\,\theta = -1$

40. A caution sign next to a roadway states that an upcoming hill has an 8% slope. An 8% slope means that there is an 8 ft rise for 100 ft of horizontal distance. At approximately what angle does the roadway rise from the horizontal?

Ⓕ 2.2° Ⓖ 4.6° Ⓗ 8.5° Ⓙ 12.5°

41. What value of θ makes the equation $2\sqrt{2}(\text{Cos}\,\theta) = -2$ true?

Ⓐ 45° Ⓑ 60° Ⓒ 135° Ⓓ 150°

CHALLENGE AND EXTEND

42. If $\text{Sin}^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$, what is the value of $\text{Csc}^{-1}(-\sqrt{2})$?

Solve each inequality for $\{\theta \mid 0 \le \theta \le 2\pi\}$.

43. $\cos\theta \le \frac{1}{2}$ **44.** $2\sin\theta - \sqrt{3} > 0$ **45.** $\tan 2\theta \ge 1$

COMMON CORE

10-5 The Law of Sines

CC.9-12.G.SRT.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.
Also **CC.9-12.G.SRT.11** (+)

Objectives
Determine the area of a triangle given side-angle-side information.

Use the Law of Sines to find the side lengths and angle measures of a triangle.

Who uses this?
Sailmakers can use sine ratios to determine the amount of fabric needed to make a sail. (See Example 1.)

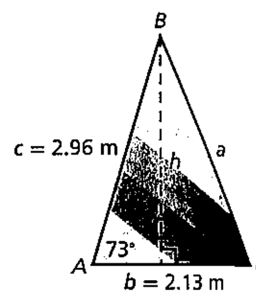
A sailmaker is designing a sail that will have the dimensions shown in the diagram. Based on these dimensions, the sailmaker can determine the amount of fabric needed.

The area of the triangle representing the sail is $\frac{1}{2}bh$. Although you do not know the value of h, you can calculate it by using the fact that $\sin A = \frac{h}{c}$, or $h = c \sin A$.

$\text{Area} = \frac{1}{2}bh$ *Write the area formula.*

$\text{Area} = \frac{1}{2}bc\sin A$ *Substitute c sin A for h.*

This formula allows you to determine the area of a triangle if you know the lengths of two of its sides and the measure of the angle between them.



Helpful Hint
An angle and the side opposite that angle are labeled with the same letter. Capital letters are used for angles, and lowercase letters are used for sides.

Know it! Note

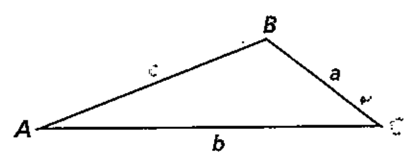
Area of a Triangle

For $\triangle ABC$,

$\text{Area} = \frac{1}{2}bc\sin A$

$\text{Area} = \frac{1}{2}ac\sin B$

$\text{Area} = \frac{1}{2}ab\sin C$



EXAMPLE 1 Determining the Area of a Triangle

Find the area of the sail shown at the top of the page. Round to the nearest tenth.

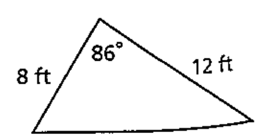
$\text{area} = \frac{1}{2}bc\sin A$ *Write the area formula.*

$= \frac{1}{2}(2.13)(2.96)\sin 73°$ *Substitute 2.13 for b, 2.96 for c, and 73° for A.*

≈ 3.014655113 *Use a calculator to evaluate the expression.*

The area of the sail is about 3.0 m^2.

CHECK IT OUT! 1. Find the area of the triangle. Round to the nearest tenth.



The area of $\triangle ABC$ is equal to $\frac{1}{2}bc\sin A$ or $\frac{1}{2}ac\sin B$ or $\frac{1}{2}ab\sin C$. By setting these expressions equal to each other, you can derive the Law of Sines.

$$\frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B = \frac{1}{2}ab\sin C$$

$$bc\sin A = ac\sin B = ab\sin C \qquad \textit{Multiply each expression by 2.}$$

$$\frac{\cancel{b}\cancel{c}\sin A}{a\cancel{b}\cancel{c}} = \frac{\cancel{a}\cancel{c}\sin B}{\cancel{a}b\cancel{c}} = \frac{\cancel{a}\cancel{b}\sin C}{\cancel{a}\cancel{b}c} \qquad \textit{Divide each expression by abc.}$$

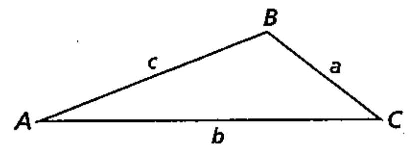
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \qquad \textit{Divide out common factors.}$$

Know it! Note

Law of Sines

For $\triangle ABC$, the Law of Sines states that

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$



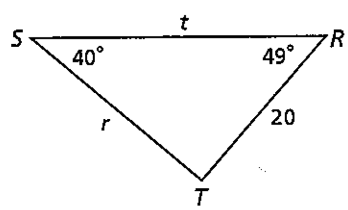
The Law of Sines allows you to solve a triangle as long as you know either of the following:

1. Two angle measures and any side length—angle-angle-side (AAS) or angle-side-angle (ASA) information
2. Two side lengths and the measure of an angle that is not between them—side-side-angle (SSA) information

EXAMPLE 2 **Using the Law of Sines for AAS and ASA**

Solve the triangle. Round to the nearest tenth.

A



Reading Math

The expression "solve a triangle" means to find the measures of all unknown angles and sides.

Step 1 Find the third angle measure.

$m\angle R + m\angle S + m\angle T = 180°$ *Triangle Sum Theorem*

$49° + 40° + m\angle T = 180°$ *Substitute 49° for $m\angle R$ and 40° for $m\angle S$.*

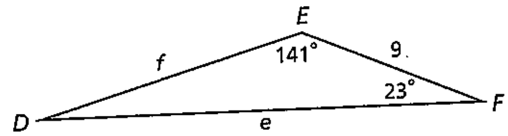
$m\angle T = 91°$ *Solve for $m\angle T$.*

Step 2 Find the unknown side lengths.

$\frac{\sin R}{r} = \frac{\sin S}{s}$	*Law of Sines*	$\frac{\sin S}{s} = \frac{\sin T}{t}$
$\frac{\sin 49°}{r} = \frac{\sin 40°}{20}$	*Substitute.*	$\frac{\sin 40°}{20} = \frac{\sin 91°}{t}$
$r\sin 40° = 20\sin 49°$	*Cross multiply.*	$t\sin 40° = 20\sin 91°$
$r = \frac{20\sin 49°}{\sin 40°}$	*Solve for the unknown side.*	$t = \frac{20\sin 91°}{\sin 40°}$
$r \approx 23.5$		$t \approx 31.1$

Solve the triangle. Round to the nearest tenth.

B



Step 1 Find the third angle measure.

$m\angle D = 180° - 141° - 23° = 16°$ *Triangle Sum Theorem*

Step 2 Find the unknown side lengths.

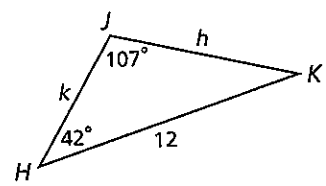
$\frac{\sin D}{d} = \frac{\sin E}{e}$ *Law of Sines* $\qquad \frac{\sin D}{d} = \frac{\sin F}{f}$

$\frac{\sin 16°}{9} = \frac{\sin 141°}{e}$ *Substitute.* $\qquad \frac{\sin 16°}{9} = \frac{\sin 23°}{f}$

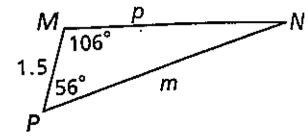
$e = \frac{9 \sin 141°}{\sin 16°} \approx 20.5 \qquad f = \frac{9 \sin 23°}{\sin 16°} \approx 12.8$

CHECK IT OUT! Solve each triangle. Round to the nearest tenth.

2a.



2b.

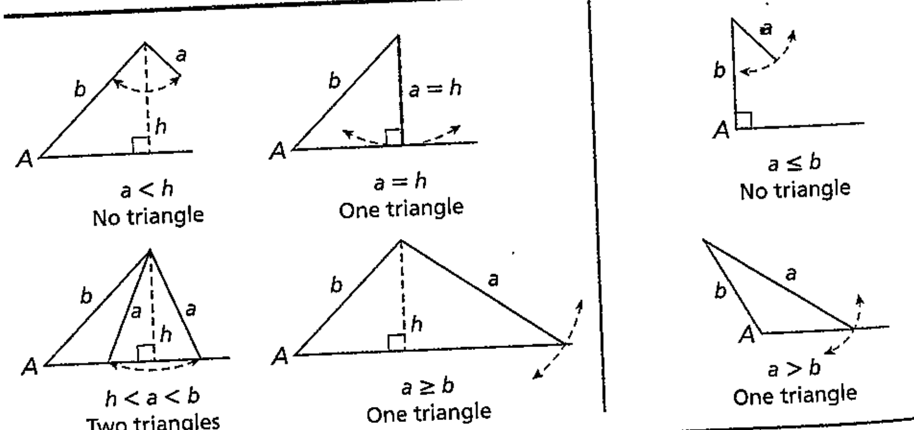


When you use the Law of Sines to solve a triangle for which you know side-side-angle (SSA) information, zero, one, or two triangles may be possible. For this reason, SSA is called the *ambiguous case*.

Ambiguous Case **Possible Triangles**

Given *a*, *b*, and m∠*A*,

∠*A* IS ACUTE.		∠*A* IS RIGHT OR OBTUSE.



Remember!

When one angle in a triangle is obtuse, the measures of the other two angles must be acute.

Know it! Note

Solving a Triangle Given *a*, *b*, and m∠*A*
1. Use the values of *a*, *b*, and m∠*A* to determine the number of possible triangles.
2. If there is one triangle, use the Law of Sines to solve for the unknowns.
3. If there are two triangles, use the Law of Sines to find $m\angle B_1$ and $m\angle B_2$. Then use these values to find the other measurements of the two triangles.

EXAMPLE 3 *Art Application*

Maggie is designing a mosaic by using triangular tiles of different shapes. Determine the number of triangles that Maggie can form using the measurements $a = 11$ cm, $b = 17$ cm, and $m\angle A = 30°$. Then solve the triangles. Round to the nearest tenth.

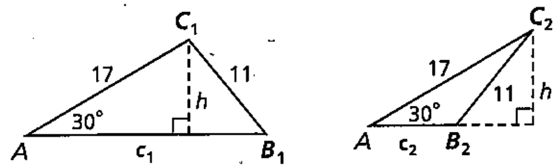
Step 1 Determine the number of possible triangles. In this case, $\angle A$ is acute. Find h.

$$\sin 30° = \frac{h}{17} \qquad \textit{sin } \theta = \frac{\textit{opp.}}{\textit{hyp.}}$$

$$h = 17\sin 30° \approx 8.5 \text{ cm} \qquad \textit{Solve for h.}$$

Because $h < a < b$, two triangles are possible.

Triangle 1 **Triangle 2**



Step 2 Determine $m\angle B_1$ and $m\angle B_2$.

$$\frac{\sin A}{a} = \frac{\sin B}{b} \qquad \textit{Law of Sines}$$

$$\frac{\sin 30°}{11} = \frac{\sin B}{17} \qquad \textit{Substitute.}$$

$$\sin B = \frac{17\sin 30°}{11} \qquad \textit{Solve for sin B.}$$

$$\sin B \approx 0.773$$

Let $\angle B_1$ represent the acute angle with a sine of 0.773. Use the inverse sine function on your calculator to determine $m\angle B_1$.

$$m\angle B_1 = \text{Sin}^{-1}\left(\frac{17\sin 30°}{11}\right) \approx 50.6°$$

Let $\angle B_2$ represent the obtuse angle with a sine of 0.773.

$$m\angle B_2 = 180° - 50.6° = 129.4° \qquad \textit{The reference angle of } \angle B_2 \textit{ is } 50.6°.$$

Step 3 Find the other unknown measures of the two triangles.

Solve for $m\angle C_1$.

$$30° + 50.6° + m\angle C_1 = 180°$$
$$m\angle C_1 = 99.4°$$

Solve for $m\angle C_2$.

$$30° + 129.4° + m\angle C_2 = 180°$$
$$m\angle C_2 = 20.6°$$

Solve for c_1.

$$\frac{\sin A}{a} = \frac{\sin C_1}{c_1} \qquad \textit{Law of Sines}$$

$$\frac{\sin 30°}{11} = \frac{\sin 99.4°}{c_1} \qquad \textit{Substitute.}$$

$$c_1 = \frac{11\sin 99.4°}{\sin 30°} \qquad \textit{Solve for the unknown side.}$$

$$c_1 \approx 21.7 \text{ cm}$$

Solve for c_2.

$$\frac{\sin A}{a} = \frac{\sin C_2}{c_2}$$

$$\frac{\sin 30°}{11} = \frac{\sin 20.6°}{c_2}$$

$$c_2 = \frac{11\sin 20.6°}{\sin 30°}$$

$$c_2 \approx 7.7 \text{ cm}$$

Helpful Hint

Because $\angle B_1$ and $\angle B_2$ have the same sine value, they also have the same reference angle.

CHECK IT OUT!

3. Determine the number of triangles Maggie can form using the measurements $a = 10$ cm, $b = 6$ cm, and $m\angle A = 105°$. Then solve the triangles. Round to the nearest tenth.

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Explain how right triangle trigonometry can be used to determine the area of an obtuse triangle.
2. Explain why using the Law of Sines when given AAS or ASA is different than when given SSA.

Know it! Note

3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, give the conditions for which the ambiguous case results in zero, one, or two triangles.

SSA: Given *a*, *b*, and m∠*A*			
Angle *A*	**0 triangles**	**1 triangle**	**2 triangles**
Obtuse			
Acute			

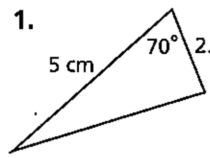
10-5 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

SEE EXAMPLE 1

Find the area of each triangle. Round to the nearest tenth.

1.


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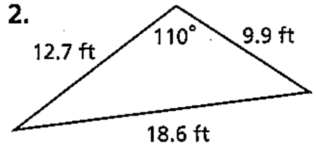

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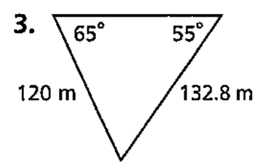
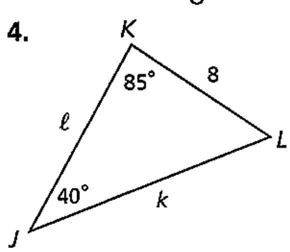


SEE EXAMPLE 2

Solve each triangle. Round to the nearest tenth.

4.


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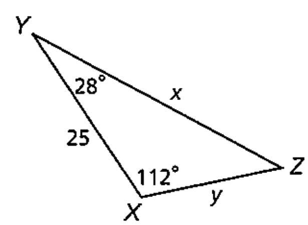

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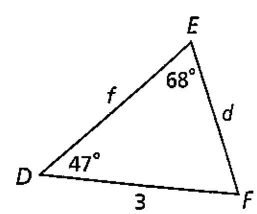

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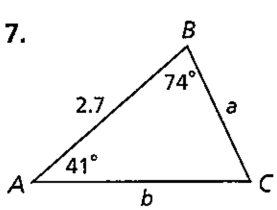

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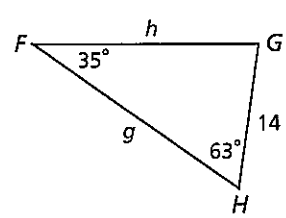

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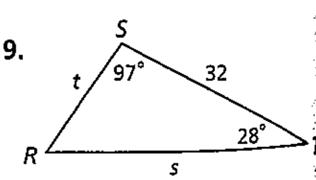


SEE EXAMPLE 3

Gardening A landscape architect is designing triangular flower beds. Determine the number of different triangles that he can form using the given measurements. Then solve the triangles. Round to the nearest tenth.

10. $a = 6$ m, $b = 9$ m, $m\angle A = 55°$
11. $a = 10$ m, $b = 4$ m, $m\angle A = 120°$
12. $a = 8$ m, $b = 9$ m, $m\angle A = 35°$
13. $a = 7$ m, $b = 6$ m, $m\angle A = 45°$

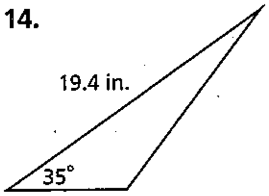
...ndent Practice	
...or ...cises	See Example
...–16	1
...–19	2
...–23	3

...a Practice
...a Practice for ...kills Practice and ...tions Practice ...es

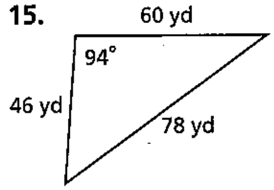
PRACTICE AND PROBLEM SOLVING

Find the area of each triangle. Round to the nearest tenth.

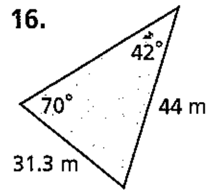
14.



15.

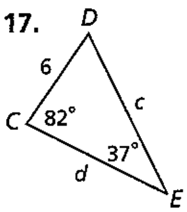


16.

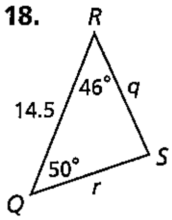


Solve each triangle. Round to the nearest tenth.

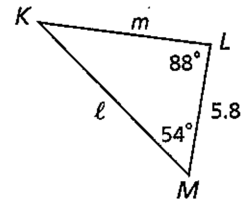
17.



18.



19.



Art **An artist is designing triangular mirrors. Determine the number of different triangles that she can form using the given measurements. Then solve the triangles. Round to the nearest tenth.**

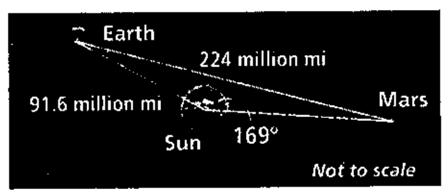
20. $a = 6$ cm, $b = 4$ cm, $m\angle A = 72°$

21. $a = 3.0$ in., $b = 3.5$ in., $m\angle A = 118°$

22. $a = 4.2$ cm, $b = 5.7$ cm, $m\angle A = 39°$

23. $a = 7$ in., $b = 3.5$ in., $m\angle A = 130°$

24. Astronomy The diagram shows the relative positions of Earth, Mars, and the Sun on a particular date. What is the distance between Mars and the Sun on this date? Round to the nearest million miles.



Use the given measurements to solve △*ABC*. Round to the nearest tenth.

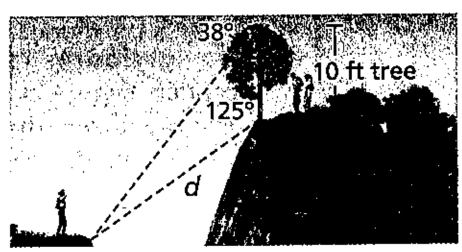
25. $m\angle A = 54°$, $m\angle B = 62°$, $a = 14$

26. $m\angle A = 126°$, $m\angle C = 18°$, $c = 3$

27. $m\angle B = 80°$, $m\angle C = 41°$, $b = 25$

28. $m\angle A = 24°$, $m\angle B = 104°$, $c = 10$

29. Rock Climbing A group of climbers needs to determine the distance from one side of a ravine to another. They make the measurements shown. To the nearest foot, what is the distance *d* across the ravine?



Determine the number of different triangles that can be formed using the given measurements. Then solve the triangles. Round to the nearest tenth.

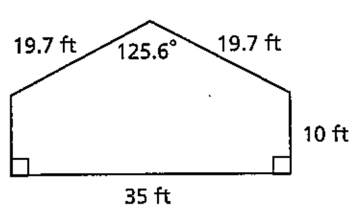
30. $m\angle C = 45°$, $b = 10$, $c = 5$

31. $m\angle B = 135°$, $b = 12$, $c = 8$

32. $m\angle A = 60°$, $a = 9$, $b = 10$

33. $m\angle B = 30°$, $a = 6$, $b = 3$

34. Painting Trey needs to paint a side of a house that has the measurements shown. What is the area of this side of the house to the nearest square foot?

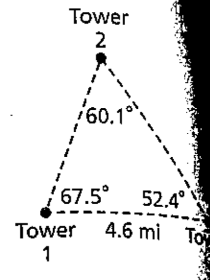


MULTI-STEP TEST PREP

35. An emergency dispatcher must determine the position of a caller reporting a fire. Based on the caller's cell phone records, she is located in the area shown.

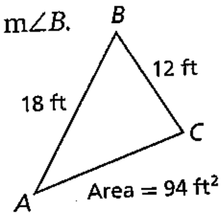
    a. To the nearest tenth of a mile, what are the unknown side lengths of the triangle?

    b. What is the area in square miles of the triangle in which the caller is located? Round to the nearest tenth.

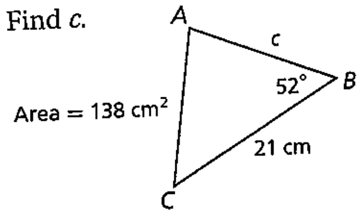


Find the indicated measurement. Round to the nearest tenth.

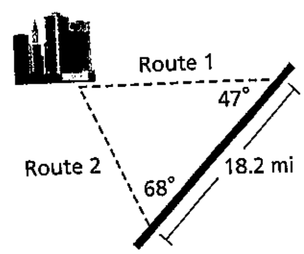
36. Find m∠B.



37. Find c.



38. **Multi-Step** A new road will be built from a town to a nearby highway. So far, two routes have been proposed. To the nearest tenth of a mile, how much shorter is route 2 than route 1?



39. **///ERROR ANALYSIS///** Below are two attempts at solving $\triangle FGH$ for g. Which is incorrect? Explain the error.

(A)

$$\frac{\sin 41°}{4.8} = \frac{\sin 74°}{g}$$

$$g = \frac{4.8 \sin 74°}{\sin 41°}$$

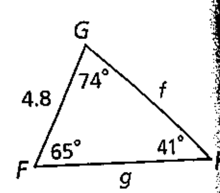
$$g \approx 7.0$$

(B)

$$\frac{\sin 74°}{4.8} = \frac{\sin 41°}{g}$$

$$g = \frac{4.8 \sin 41°}{\sin 74°}$$

$$g \approx 3.3$$



LINK

Navigation

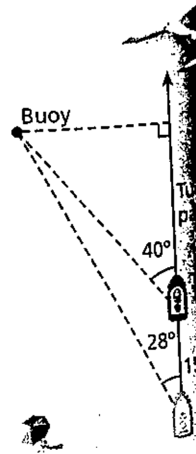
Ship captains rely heavily on GPS technology. GPS stands for "Global Positioning System," a navigation system based on trigonometry and the use of satellites. The GPS can be used to determine information such as latitude and longitude.

40. **Navigation** As a tugboat travels along a channel, the captain sights a buoy at an angle of 28° to the boat's path. The captain continues on the same course for a distance of 1500 m and then sights the same buoy at an angle of 40°.

    a. To the nearest meter, how far is the tugboat from the buoy at the second sighting?

    b. To the nearest meter, how far was the tugboat from the buoy when the captain first sighted the bouy?

    c. **What if...?** If the tugboat continues on the same course, what is the closest that it will come to the buoy? Round to the nearest meter.



41. **Critical Thinking** How can you tell, without using the Law of Sines, that a triangle cannot be formed by using the measurements m∠A = 92°, m∠B = 104°, and a = 18?

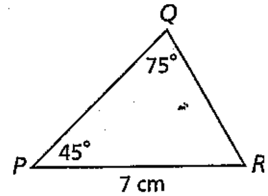
42. **Write About It** Explain how to solve a triangle when angle-angle-side (AAS) information is known.

TEST PREP

43. What is the area of $\triangle PQR$ to the nearest tenth of a square centimeter?

Ⓐ $2.4\ cm^2$ Ⓒ $23.5\ cm^2$

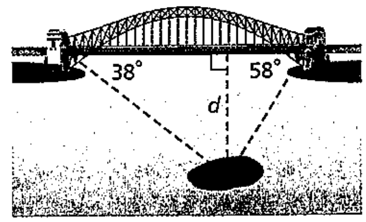
Ⓑ $15.5\ cm^2$ Ⓓ $40.1\ cm^2$



44. A bridge is 325 m long. From the west end, a surveyor measures the angle between the bridge and an island to be 38°. From the east end, the surveyor measures the angle between the bridge and the island to be 58°. To the nearest meter, what is the distance *d* between the bridge and the island?

Ⓕ 171 m Ⓗ 217 m

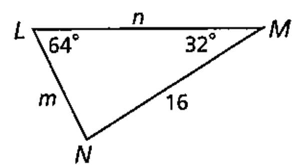
Ⓖ 201 m Ⓙ 277 m



45. **Short Response** Examine $\triangle LMN$ at right.

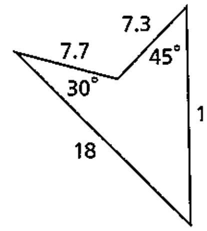
a. Write an expression that can be used to determine the value of *m*.

b. Is there more than one possible triangle that can be constructed from the given measurements? Explain your answer.



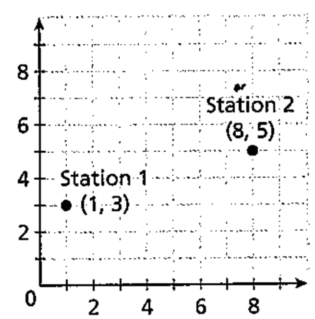
CHALLENGE AND EXTEND

46. What is the area of the quadrilateral at right to the nearest square unit?



47. **Critical Thinking** The lengths of two sides of a triangle are $a = 3$ and $b = 2\sqrt{3}$. For what values of $m\angle A$ do two solutions exist when you solve the triangle by using the Law of Sines?

48. **Multi-Step** The map shows the location of two ranger stations. Each unit on the map represents 1 mile. A ranger at station 1 saw a meteor that appeared to land about 72° north of east. A ranger at station 2 saw the meteor appear to land about 45° north of west. Based on this information, about how many miles from station 1 did the meteor land? Explain how you determined your answer.



COMMON CORE

10-6 The Law of Cosines

CC.9-12.G.SRT.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.
Also **CC.9-12.G.SRT.11 (+)**

Objectives

Use the Law of Cosines to find the side lengths and angle measures of a triangle.

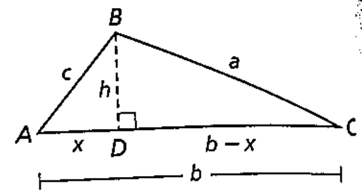
Use Heron's Formula to find the area of a triangle.

Who uses this?

Trapeze artists can use the Law of Cosines to determine whether they can perform stunts safely. (See Exercise 27.)

You learned to solve triangles by using the Law of Sines. However, the Law of Sines cannot be used to solve triangles for which side-angle-side (SAS) or side-side-side (SSS) information is given. Instead, you must use the Law of Cosines.

To derive the Law of Cosines, draw $\triangle ABC$ with altitude $\overline{BD}$. If x represents the length of $\overline{AD}$, the length of $\overline{DC}$ is $b - x$.



Write an equation that relates the side lengths of $\triangle DBC$.

$a^2 = (b - x)^2 + h^2$ — *Pythagorean Theorem*

$a^2 = b^2 - 2bx + x^2 + h^2$ — *Expand* $(b - x)^2$.

$a^2 = b^2 - 2bx + c^2$ — *In* $\triangle ABD$, $c^2 = x^2 + h^2$. *Substitute* c^2 *for* $x^2 + h^2$.

$a^2 = b^2 - 2b(c\cos A) + c^2$ — *In* $\triangle ABD$, $\cos A = \frac{x}{c}$, *or* $x = c\cos A$. *Substitute* $c\cos A$ *for* x.

$a^2 = b^2 + c^2 - 2bc\cos A$

The previous equation is one of the formulas for the Law of Cosines.

Know it! Note

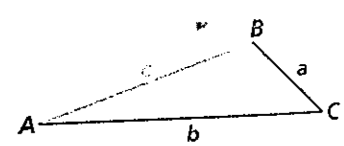
Law of Cosines

For $\triangle ABC$, the Law of Cosines states that

$a^2 = b^2 + c^2 - 2bc\cos A.$

$b^2 = a^2 + c^2 - 2ac\cos B.$

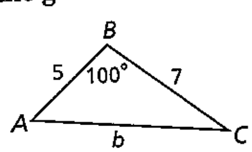
$c^2 = a^2 + b^2 - 2ab\cos C.$



EXAMPLE 1 **Using the Law of Cosines**

Use the given measurements to solve $\triangle ABC$. Round to the nearest tenth.

A



Step 1 Find the length of the third side.

$b^2 = a^2 + c^2 - 2ac\cos B$ — *Law of Cosines*

$b^2 = 7^2 + 5^2 - 2(7)(5)\cos 100°$ — *Substitute.*

$b^2 \approx 86.2$ — *Use a calculator to simplify.*

$b \approx 9.3$ — *Solve for the positive value of b.*

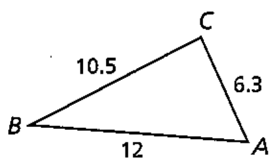
Step 2 Find an angle measure.

$\frac{\sin A}{a} = \frac{\sin B}{b}$	*Law of Sines*
$\frac{\sin A}{7} = \frac{\sin 100°}{9.3}$	*Substitute.*
$\sin A = \frac{7 \sin 100°}{9.3}$	*Solve for sin A.*
$m\angle A = \text{Sin}^{-1}\left(\frac{7 \sin 100°}{9.3}\right) \approx 47.8°$	*Solve for m∠A.*

Step 3 Find the third angle measure.

$47.8° + 100° + m\angle C \approx 180°$	*Triangle Sum Theorem*
$m\angle C \approx 32.2°$	*Solve for m∠C.*

B



Step 1 Find the measure of the largest angle, ∠C.

$c^2 = a^2 + b^2 - 2ab\cos C$	*Law of Cosines*
$12^2 = 10.5^2 + 6.3^2 - 2(10.5)(6.3)\cos C$	*Substitute.*
$\cos C \approx 0.0449$	*Solve for cos C.*
$m\angle C \approx \text{Cos}^{-1}(0.0449) \approx 87.4°$	*Solve for m∠C.*

Remember!

The largest angle of a triangle is the angle opposite the longest side.

Step 2 Find another angle measure.

$b^2 = a^2 + c^2 - 2ac\cos B$	*Law of Cosines*
$6.3^2 = 10.5^2 + 12^2 - 2(10.5)(12)\cos B$	*Substitute.*
$\cos B \approx 0.8514$	*Solve for cos B.*
$m\angle B \approx \text{Cos}^{-1}(0.8514) \approx 31.6°$	*Solve for m∠B.*

Step 3 Find the third angle measure.

$m\angle A + 31.6° + 87.4° \approx 180°$	*Triangle Sum Theorem*
$m\angle A \approx 61.0°$	*Solve for m∠A.*

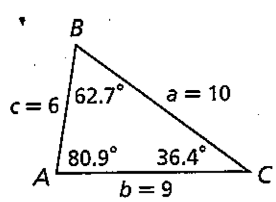
CHECK IT OUT! **Use the given measurements to solve △ABC. Round to the nearest tenth.**

1a. $b = 23$, $c = 18$, $m\angle A = 173°$ **1b.** $a = 35$, $b = 42$, $c = 50.3$

Student to Student *Solving Triangles*

Stefan Maric
Wylie High School

If I solve a triangle using the Law of Sines, I like to use the Law of Cosines to check my work. I used the Law of Sines to solve the triangle below.



I can check that the length of side b really is 9 by using the Law of Cosines.

$b^2 = a^2 + c^2 - 2ac\cos B$

9^2	$10^2 + 6^2 - 2(10)(6)\cos 62.7°$
81	81.0 ✔

The Law of Cosines shows that I was right.

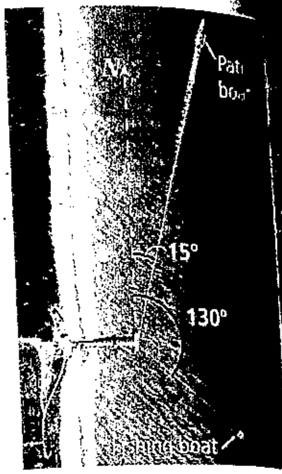
EXAMPLE 2

Problem-Solving Application

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

A coast guard patrol boat and a fishing boat leave a dock at the same time on the courses shown. The patrol boat travels at a speed of 12 nautical miles per hour (12 knots), and the fishing boat travels at a speed of 5 knots. After 3 hours, the fishing boat sends a distress signal picked up by the patrol boat. If the fishing boat does not drift, how long will it take the patrol boat to reach it at a speed of 12 knots?



1 Understand the Problem

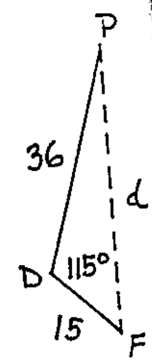
The answer will be the number of hours that the patrol boat needs to reach the fishing boat.

List the important information:

- The patrol boat's speed is 12 knots. Its direction is 15° east of north.
- The fishing boat's speed is 5 knots. Its direction is 130° east of north.
- The boats travel 3 hours before the distress call is given.

2 Make a Plan

Determine the angle between the boats' courses and the distance that each boat travels in 3 hours. Use this information to draw and label a diagram. Then use the Law of Cosines to find the distance d between the boats at the time of the distress call. Finally, determine how long it will take the patrol boat to travel this distance.



3 Solve

Step 1 Draw and label a diagram.

The angle between the boats' courses is $130° - 15° = 115°$. In 3 hours, the patrol boat travels $3(12) = 36$ nautical miles and the fishing boat travels $3(5) = 15$ nautical miles.

Step 2 Find the distance d between the boats.

$d^2 = p^2 + f^2 - 2pf\cos D$	*Law of Cosines*
$d^2 = 15^2 + 36^2 - 2(15)(36)\cos 115°$	*Substitute 15 for p, 36 for f, and 115° for D.*
$d^2 \approx 1977.4$	*Use a calculator to simplify.*
$d \approx 44.5$	*Solve for the positive value of d.*

Step 3 Determine the number of hours.

The patrol boat must travel about 44.5 nautical miles to reach the fishing boat. At a speed of 12 nautical miles per hour, it will take the patrol boat $\frac{44.5}{12} \approx 3.7$ hours to reach the fishing boat.

4 Look Back

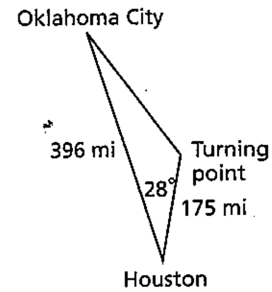
To reach the fishing boat, the patrol boat will have to travel a greater distance than it did during the first 3 hours of its trip. Therefore, it makes sense that it will take the patrol boat longer than 3 hours to reach the fishing boat. An answer of 3.7 hours seems reasonable.

Helpful Hint

There are two solutions to $d^2 = 1977.4$. One is positive, and one is negative. Because d represents a distance, the negative solution can be disregarded.

CHECK IT OUT!

2. A pilot is flying from Houston to Oklahoma City. To avoid a thunderstorm, the pilot flies 28° off of the direct route for a distance of 175 miles. He then makes a turn and flies straight on to Oklahoma City. To the nearest mile, how much farther than the direct route was the route taken by the pilot?



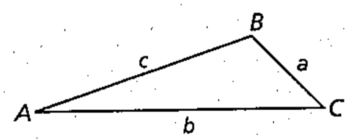
The Law of Cosines can be used to derive a formula for the area of a triangle based on its side lengths. This formula is called Heron's Formula.

Know it! Note

Heron's Formula

For $\triangle ABC$, where s is half of the perimeter of the triangle, or $\frac{1}{2}(a + b + c)$,

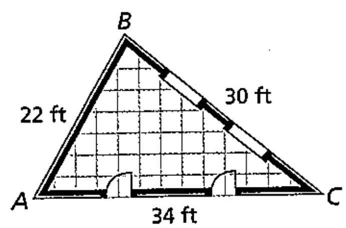
Area $= \sqrt{s(s - a)(s - b)(s - c)}$



EXAMPLE 3

Architecture Application

A blueprint shows a reception area that has a triangular floor with sides measuring 22 ft, 30 ft, and 34 ft. What is the area of the floor to the nearest square foot?



Step 1 Find the value of s.

$s = \frac{1}{2}(a + b + c)$ — *Use the formula for half of the perimeter.*

$s = \frac{1}{2}(30 + 34 + 22) = 43$ — *Substitute 30 for a, 34 for b, and 22 for c.*

Step 2 Find the area of the triangle.

$A = \sqrt{s(s - a)(s - b)(s - c)}$ — *Heron's Formula*

$A = \sqrt{43(43 - 30)(43 - 34)(43 - 22)}$ — *Substitute 43 for s.*

$A \approx 325$ — *Use a calculator to simplify.*

The area of the floor is 325 ft^2.

Check Find the measure of the largest angle, $\angle B$.

$b^2 = a^2 + c^2 - 2ac\cos B$ — *Law of Cosines*

$34^2 = 30^2 + 22^2 - 2(30)(22)\cos B$ — *Substitute.*

$\cos B \approx 0.1727$ — *Solve for cos B.*

$m\angle B \approx 80.1°$ — *Solve for m∠B.*

Find the area of the triangle by using the formula area $= \frac{1}{2}ac\sin B$.

area $= \frac{1}{2}(30)(22)\sin 80.1° \approx 325$ ft^2 ✔

CHECK IT OUT!

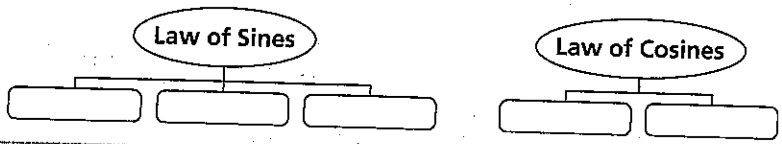
3. The surface of a hotel swimming pool is shaped like a triangle with sides measuring 50 m, 28 m, and 30 m. What is the area of the pool's surface to the nearest square meter?

THINK AND DISCUSS

1. Explain why you cannot solve a triangle if you are given only angle-angle-angle (AAA) information.
2. Describe the steps that you could use to find the area of a triangle by using Heron's Formula when you are given side-angle-side (SAS) information.

Know it! Note

3. **GET ORGANIZED** Copy and complete the graphic organizer. List the types of triangles that can be solved by using each law. Consider the following types of triangles: ASA, AAS, SAS, SSA, and SSS.



10-6 Exercises

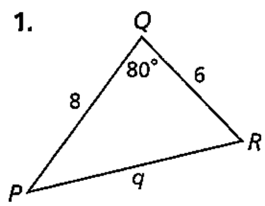
Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

SEE EXAMPLE 1

Use the given measurements to solve each triangle. Round to the nearest tenth.

1.



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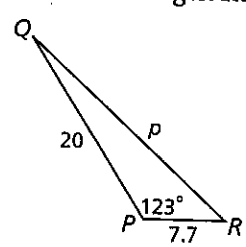

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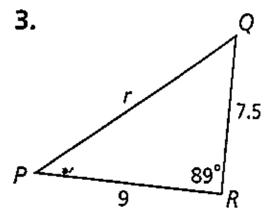

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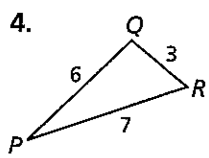

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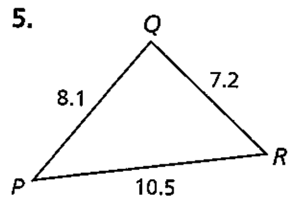

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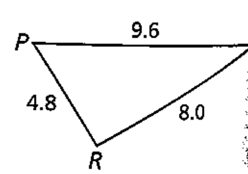


SEE EXAMPLE 2

7. **Recreation** A triangular hiking trail is being built in the area shown. At an average walking speed of 2 m/s, how many minutes will it take a hiker to make a complete circuit around the triangular trail? Round to the nearest minute.

SEE EXAMPLE 3

8. **Agriculture** A triangular wheat field has side lengths that measure 410 ft, 500 ft, and 420 ft. What is the area of the field to the nearest square foot?

ependent Practice	
For Exercises	See Example
9–14	1
15	2
16	3

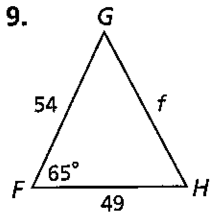
tra Practice

Extra Practice for ...Skills Practice and ...lications Practice ...rcises.

PRACTICE AND PROBLEM SOLVING

Use the given measurements to solve each triangle. Round to the nearest tenth.

9.



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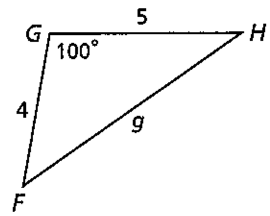

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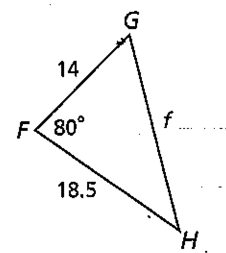

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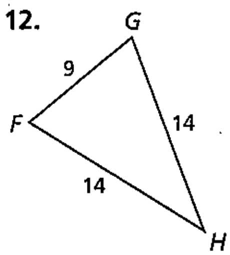

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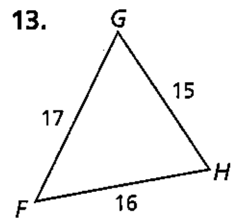

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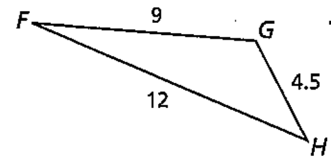
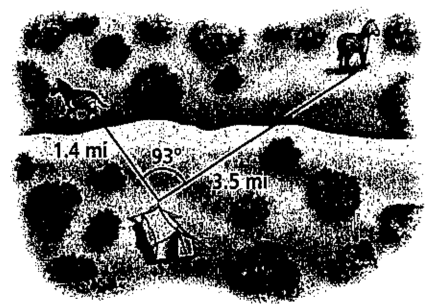


15. Ecology An ecologist is studying a pair of zebras fitted with radio-transmitter collars. One zebra is 1.4 mi from the ecologist, and the other is 3.5 mi from the ecologist. To the nearest tenth of a mile, how far apart are the two zebras?



16. Art How many square meters of fabric are needed to make a triangular banner with side lengths of 2.1 m, 1.5 m, and 1.4 m? Round to the nearest tenth.

Use the given measurements to solve $\triangle ABC$. Round to the nearest tenth.

17. $m\angle A = 120°$, $b = 16$, $c = 20$

18. $m\angle B = 78°$, $a = 6$, $c = 4$

19. $m\angle C = 96°$, $a = 13$, $b = 9$

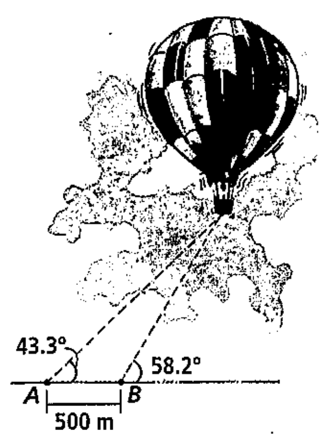
20. $a = 14$, $b = 9$, $c = 10$

21. $a = 5$, $b = 8$, $c = 6$

22. $a = 30$, $b = 26$, $c = 35$

23. Commercial Art A graphic artist is asked to draw a triangular logo with sides measuring 15 cm, 18 cm, and 20 cm. If she draws the triangle correctly, what will be the measures of its angles to the nearest degree?

24. Aviation The course of a hot-air balloon takes the balloon directly over points A and B, which are 500 m apart. Several minutes later, the angle of elevation from an observer at point A to the balloon is 43.3°, and the angle of elevation from an observer at point B to the balloon is 58.2°. To the nearest meter, what is the balloon's altitude?



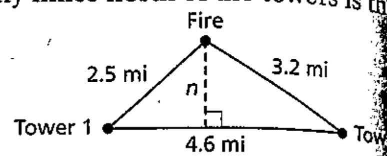
25. Multi-Step A student pilot takes off from a local airstrip and flies 70° south of east for 160 miles. The pilot then changes course and flies due north for another 80 miles before turning and flying directly back to the airstrip.

a. How many miles is the third stage of the pilot's flight? Round to the nearest mile.

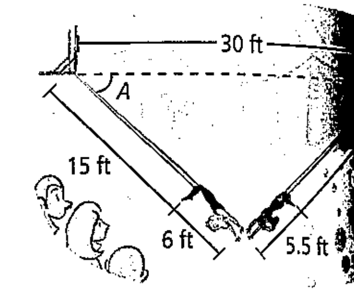
b. To the nearest degree, what angle does the pilot turn the plane through in order to fly the third stage?

MULTI-STEP TEST PREP

26. Phone records indicate that a fire is located 2.5 miles from one cell phone tower and 3.2 miles from a second cell phone tower.
    a. To the nearest degree, what are the measures of the angles of the triangle shown in the diagram?
    b. Tower 2 is directly east of tower 1. How many miles north of the towers is the fire? This distance is represented by n in the diagram.



27. **Entertainment** Two performers hang by their knees from trapezes, as shown.
    a. To the nearest degree, what acute angles A and B must the cords of each trapeze make with the horizontal if the performer on the left is to grab the wrists of the performer on the right and pull her away from her trapeze?
    b. **What if...?** Later, the performer on the left grabs the trapeze of the performer on the right and lets go of his trapeze. To the nearest degree, what angles A and B must the cords of each trapeze make with the horizontal for this trick to work?

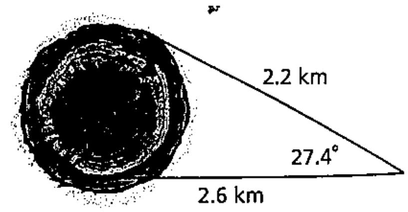


Find the area of the triangle with the given side lengths. Round to the nearest tenth.

28. 15 in., 18 in., 24 in.
29. 30 cm, 35 cm, 47 cm
30. 28 m, 37 m, 33 m
31. 3.5 ft, 5 ft, 7.5 ft

32. **Estimation** The adjacent sides of a parallelogram measure 3.1 cm and 3.9 cm. The measures of the acute interior angles of the parallelogram are 58°. Estimate the lengths of the diagonals of the parallelogram without using a calculator, and explain how you determined your estimates.

33. **Surveying** Barrington Crater in Arizona was produced by the impact of a meteorite. Based on the measurements shown, what is the diameter d of Barrington Crater to the nearest tenth of a kilometer?



34. **Travel** The table shows the distances between three islands in Hawaii. To the nearest degree, what is the angle between each pair of islands in relation to the third island?

Distances Between Islands (mi)

	Kauai	Molokai	Lanai
Kauai	0	155.7	174.8
Molokai	155.7	0	26.1
Lanai	174.8	26.1	0

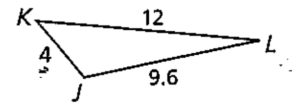
35. **Critical Thinking** Use the Law of Cosines to explain why $c^2 = a^2 + b^2$ for $\triangle ABC$, where $\angle C$ is a right angle.

36. **Critical Thinking** Can the value of s in Heron's Formula ever be less than the length of the longest side of a triangle? Explain.

37. **Write About It** Describe two different methods that could be used to solve a triangle when given side-side-side (SSS) information.

TEST PREP

38. What is the approximate measure of ∠K in the triangle shown?



Ⓐ 30° Ⓒ 54°

Ⓑ 45° Ⓓ 60°

39. For △RST with side lengths r, s, and t, which equation can be used to determine r?

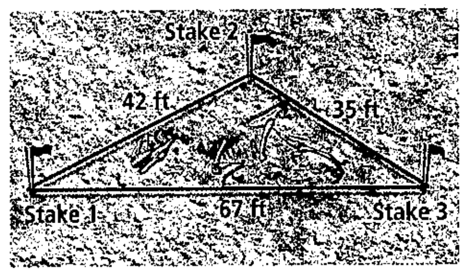
Ⓕ $r = \sqrt{s^2 + t^2 - 2st \sin R}$

Ⓗ $r = \sqrt{s^2 + t^2 - 2st \cos R}$

Ⓖ $r = \sqrt{s^2 - t^2 - 2st \sin R}$

Ⓙ $r = \sqrt{s^2 - t^2 - 2st \cos R}$

40. A team of archaeologists wants to dig for fossils in a triangular area marked by three stakes. The distances between the stakes are shown in the diagram. Which expression represents the dig area in square feet?



Ⓐ $\sqrt{72(30)(37)(5)}$

Ⓑ $\sqrt{48(6)(13)(19)}$

Ⓒ $\sqrt{144(42)(35)(67)}$

Ⓓ $\sqrt{144(102)(109)(77)}$

CHALLENGE AND EXTEND

41. Abby uses the Law of Cosines to find m∠A when $a = 2$, $b = 3$, and $c = 5$. The answer she gets is 0°. Did she make an error? Explain.

42. Geometry What are the angle measures of an isosceles triangle whose base is half as long as its congruent legs? Round to the nearest tenth.

43. Use the figure shown to solve for x. Round to the nearest tenth.