Infinity & Beyond

$$\int_0 X^2 \, dx$$

AP Calculus BC

SUMMER REVIEW SUPPLEMENTARY PACKET FOR BC STUDENTS

For students entering ***CALCULUS*** (AP Calculus BC)
Valley Christian High School

Name: ____

Course: ________________________ Period ____________________

<u>This is Summer Homework Packet #2. BC Students are also required to do Packet #1 homework, common to both AB and BC Students.</u>

The problems in this packet are designed to help you review pre-requisite skills that will help you succeed in AP Calculus C. The topics: infinite series, parametric equations and polar functions are covered in Semester 2 as part of AP Calculus C topics. You'll find that the problems in this packet are designed to expose topics that you learned in Algebra 2 or Trigonometry. (Does not cover calculus). We will learn the calculus in class! The purpose of the summer packet is to pull in the skills you learned 1 or 2 years ago from your previous math classes. If you find you do not remember series and sequences, polar and parametric graphs, then feel free to spend a bit of time googling those topics during the summer.

Bring in both summer packets (showing work!) due on the first day

Enjoy your summer! Any questions? Email me at dshak@vcs.net or diana@shaknet.com.

~ *Mrs. Shak*

Summation and Limits

Sum of Constant

$$\sum_{i=1}^{n} c = cn$$

Sum with coefficient

$$\sum_{i=1}^{n} ka_i = k\sum_{i=1}^{n} a_i$$

Sum of a Sum/Difference

$$\sum_{i=1}^{n}\left(a_i \pm b_i\right) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$$

Sum of Linear Term

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Sum of Squared Term

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Sum of Cubed Term

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

Limits to Infinity

$$\lim_{n\to\infty}\frac{1}{n} = 0 \qquad \lim_{n\to\infty}\frac{an^c}{bn^c} = \frac{a}{b}$$

Examples:

$$\sum_{i=1}^{8} i = \frac{8(9)}{2} = 36 \qquad \sum_{i=1}^{3} i^2 = \frac{3(4)(7)}{6} = 14 \qquad \sum_{i=1}^{5} i^3 = \frac{5^2 6^2}{4} = 225$$

Evaluate the sum:

1. $\displaystyle\sum_{i=1}^{5}\frac{i}{2} =$

2. $\displaystyle\sum_{i=1}^{5}\frac{3}{4}i^2 =$

Evaluate the sum S(n) in terms of n, then find $\lim_{n\to\infty} S(n)$ **:**

3. S(n) = $\displaystyle\sum_{i=1}^{n}\frac{i}{n^2} =$

$\displaystyle\lim_{n\to\infty} S(n) =$

4. S(n) = $\displaystyle\sum_{i=1}^{n}\frac{i+2}{n^2} =$

$\displaystyle\lim_{n\to\infty} S(n) =$

5. S(n) = $\displaystyle\sum_{i=1}^{n}\left[\left(1+\frac{i}{n}\right)^2\left(\frac{1}{n}\right)\right] =$

$\displaystyle\lim_{n\to\infty} S(n) =$

6. S(n) = $\displaystyle\sum_{i=1}^{n}\left[3-2\left(\frac{i}{n}\right)\right]\left(\frac{1}{n}\right) =$

$\displaystyle\lim_{n\to\infty} S(n) =$

Parametric Equations

Curves are often represented by parametric equations, where a third variable (or parameter) is introduced to give more information about the curve. A common parameter might be *t* or θ .
To convert a set of parametric equations into rectangular equations, solve for the parameter (*t*) in terms of one variable, substitute for *t* in the other equation. Rewrite the new equation, eliminating the *t*.

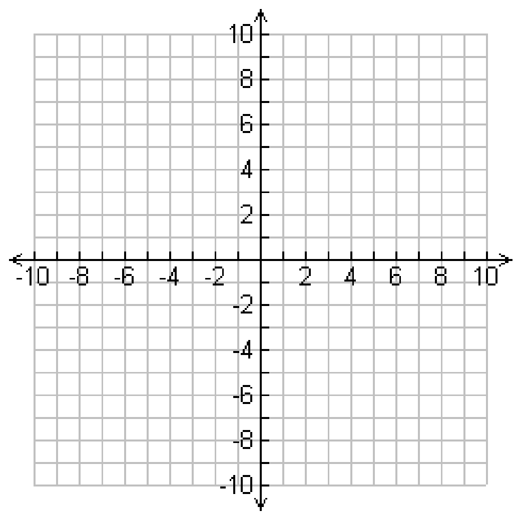
Example: $x = 2t - 3$, $y = 3t + 1$

Solve for t: $t = \frac{x+3}{2}$ **Substitute and eliminate t:** $y = 3\left(\frac{x+3}{2}\right) + 1 = \frac{3x}{2} + \frac{11}{2}$

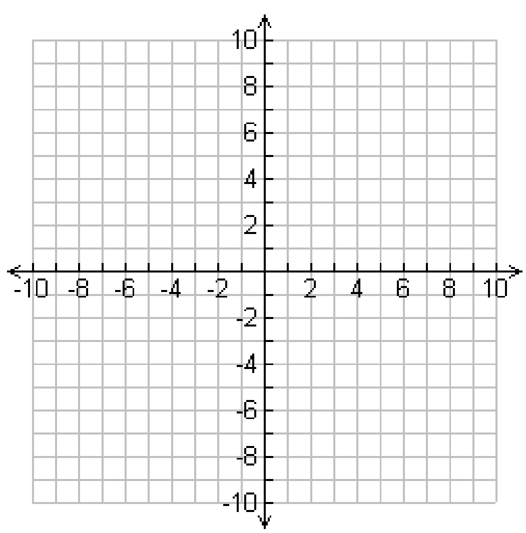
You can now graph the parametric equation by graphing the new rectangular equation. Draw in direction arrows to indicate which way the curve (or line) is moving as t increases. For parametric equations involving trig functions, make use of identities like $\sin^2\theta + \cos^2\theta = 1$ to eliminate θ

Rewrite the following parametric equations into rectangular equation. Then, sketch the curve and indicate the direction of the curve (via direction arrows).

7. $x = t + 1$
$y = t^2$

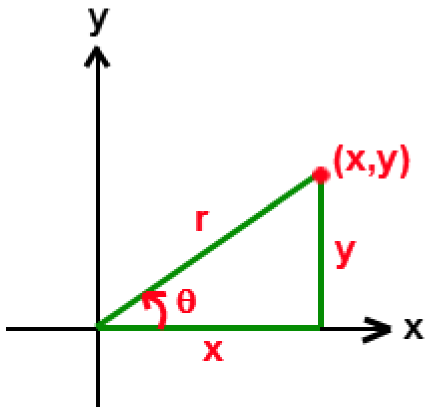


8. $x = 4 + 2\cos\theta$
$y = -1 + \sin\theta$



Polar Coordinates and Polar Equations

The polar coordinates (r, θ) are related to the rectangular coordinates (x,y) as follows. You should be able to deduce these relationships by examining the following right triangle.



$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$x^2 + y^2 = r^2$$
$$\tan\theta = \frac{y}{x}$$

Convert the following polar coordinates to rectangular coordinates:

9. $\left(8, \frac{\pi}{2}\right) =$

10. $\left(-2, \frac{5\pi}{3}\right) =$

11. $\left(-4, \frac{-3\pi}{4}\right) =$

12. $\left(0, \frac{-7\pi}{6}\right)$

Given the following rectangular coordinates, find 2 sets of polar coordinates of the point for $0 \le \theta \le 2\pi$.

13. $(4,6) =$

14. $(-3,4) =$

15. $\left(-\sqrt{3}, -\sqrt{3}\right) =$

16. $(3,-1) =$

Convert the rectangular equation to polar form:

17. $x^2 + y^2 = 9$

18. $3x - y + 2 = 0$

19. $y^2 - 8x - 16 = 0$

Convert the polar equation to rectangular form:

20. $r = 4\sin\theta$

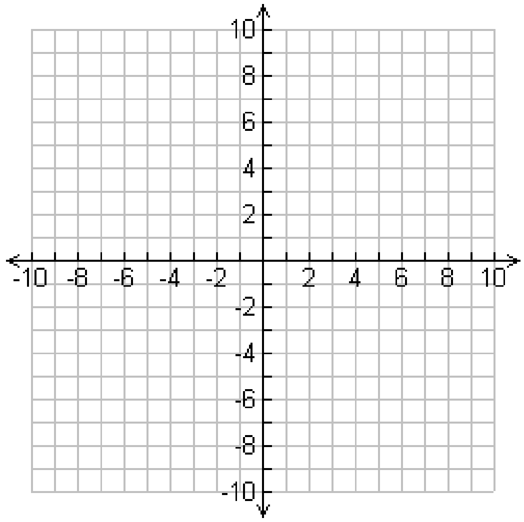
21. $\theta = \dfrac{\pi}{6}$

22. $r = \dfrac{6}{2 - 3\sin\theta}$

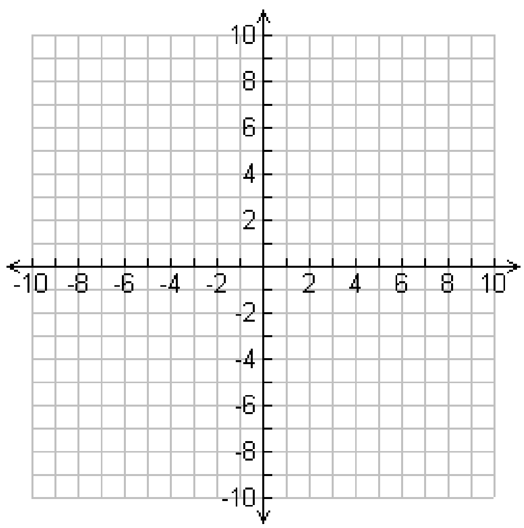
Sketch the graph of the polar equation by hand, i.e. without using a graphing calculator.

NOTE: You can use the calculator afterwards to verify that your graph is correct, but I do expect you to be able to sketch these graphs by hand (i.e. you may be asked to sketch polar graphs on a no calculator quiz).

23. $r = 2\cos(3\theta)$



24. $r = 2 + 4\sin\theta$



25. $r = 3(1 - \cos\theta)$