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BENCHMARK 2

A. Solving Equations in one Variable (pp. 19–22)

To solve an equation in one variable, isolate the variable on one side of the equation. The following examples illustrate different ways to isolate the variable.

1. Solve an Equation Using Addition or Subtraction

Vocabulary

Inverse operations Two operations that undo each other, such as addition and subtraction or multiplication and division.

Equivalent equations Equations that have the same solution(s).

EXAMPLE **Use addition or subtraction to solve the equation.**

a. $x + 9 = 3$ **b.** $x - 5 = 2$ **c.** $x + 4.1 = 6$

Solution:

Be sure to subtract (or add) the same number from each side, so that the new equation is *equivalent* to the original equation.

a.

$x + 9 = 3$ — **Write original equation.**

$x + 9 - 9 = 3 - 9$ — **Use subtraction property of equality. Subtract 9 from each side.**

$x = -6$ — **Simplify.**

The solution is -6.

b.

$x - 5 = 2$ — **Write original equation.**

$x - 5 + 5 = 2 + 5$ — **Use addition property of equality. Add 5 to each side.**

$x = 7$ — **Simplify.**

The solution is 7.

c.

$x + 4.1 = 6$ — **Write original equation.**

$x + 4.1 - 4.1 = 6 - 4.1$ — **Subtract 4.1 from each side.**

$x = 1.9$ — **Simplify.**

The solution is 1.9.

PRACTICE **Use addition or subtraction to solve the equation.**

1. $x + 5 = 4$ **2.** $c - 3 = 8$ **3.** $t + 6 = 10$

2. Solve an Equation Using Multiplication or Division

EXAMPLE **Use multiplication or division to solve the equation.**

a. $\frac{x}{6} = 3$ **b.** $-7x = -49$ **c.** $-\frac{3}{8}x = 5$

Solution:

a.

$\frac{x}{6} = 3$ — **Write original equation.**

$6 \cdot \frac{x}{6} = 6 \cdot 3$ — **Multiply each side by 6.**

$x = 18$ — **Simplify.**

The solution is 18.

BENCHMARK 2 A. Solving Equations

BENCHMARK 2

b. $-7x = -49$ Write original equation.

$\frac{-7x}{-7} = \frac{-49}{-7}$ Divide each side by -7.

$x = 7$ Simplify.

The solution is 7.

c. $-\frac{3}{8}x = 5$ Write original equation.

$-\frac{8}{3}\left(-\frac{3}{8}x\right) = -\frac{8}{3}(5)$ Multiply each side by the reciprocal $-\frac{8}{3}$.

$x = -\frac{40}{3}$ Simplify.

The solution is $-\frac{40}{3}$.

Recall that the product of a number and its *reciprocal* is 1.

PRACTICE **Use multiplication or division to solve the equation.**

4. $\frac{r}{10} = 2$ **5.** $4q = 32$ **6.** $\frac{a}{9} = -3$

3. Solve a Two-Step Equation

Vocabulary **Order of operations** The rules for evaluating an expression involving more than one operation.

EXAMPLE **Solve the equation.**

a. $7x + 1 = 4$ **b.** $\frac{x}{5} - 10 = 20$

Solution:

a. $7x + 1 = 4$ Write original equation.

$7x + 1 - 1 = 4 - 1$ Subtract 1 from each side.

$7x = 3$ Simplify.

$\frac{7x}{7} = \frac{3}{7}$ Divide each side by 7.

$x = \frac{3}{7}$ Simplify.

The solution is $\frac{3}{7}$.

b. $\frac{x}{5} - 10 = 20$ Write original equation.

$\frac{x}{5} - 10 + 10 = 20 + 10$ Add 10 to each side.

$\frac{x}{5} = 30$ Simplify.

$5 \cdot \frac{x}{5} = 5 \cdot 30$ Multiply each side by 5.

$x = 150$ Simplify.

The solution is 150.

PRACTICE **Solve the equation.**

7. $3 + 4x = 11$ **8.** $7.5a - 10 = -32.5$ **9.** $\frac{t}{8} + 6 = 3$

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BENCHMARK 2
(Chapters 3 and 4)

4. Solve Multi-Step Equations

EXAMPLE **Solve $4x + 3(x - 5) = -12$.**

Solution:

Distributive Property: $a(b + c) = ab + ac$; $a(b - c) = ab - ac$

$4x + 3(x - 5) = -12$	Write original equation.
$4x + 3x - 15 = -12$	Eliminate the parentheses by using the distributive property.
$7x - 15 = -12$	Combine like terms.
$7x - 15 + 15 = -12 + 15$	Add 15 to each side.
$7x = 3$	Simplify.
$\frac{7x}{7} = \frac{3}{7}$	Divide each side by 7.
$x = \frac{3}{7}$	Simplify.

The solution is $\frac{3}{7}$.

PRACTICE **Solve the equation.**

10. $-2(4a + 5) - 6a = 10$ **11.** $6 + 3(n - 7) = 12$ **12.** $\frac{2}{3}(6g + 1) + \frac{1}{3} = -19$

5. Solve Equations with Variables on Both Sides

EXAMPLE **Solve $x - 4 = 3x + 8$.**

Solution:

$x - 4 = 3x + 8$	Write original equation.
$x - 4 - 3x = 3x + 8 - 3x$	Subtract $3x$ from each side.
$-2x - 4 = 8$	Simplify each side.
$-2x - 4 + 4 = 8 + 4$	Add 4 to each side.
$-2x = 12$	Simplify.
$\frac{-2x}{-2} = \frac{12}{-2}$	Divide each side by -2.
$x = -6$	Simplify.

The solution is -6.

You could also begin solving the equation by adding 4 to each side to obtain $x = 3x + 12$. You will get the same solution when you finish solving for x.

PRACTICE **Solve the equation.**

13. $8t - 10 = 5 + 3t$ **14.** $-9 - 4h = -2h + 3$ **15.** $12d + 4 = 6 - d$

BENCHMARK 2
A. Solving Equations

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BENCHMARK 2
(Chapters 3 and 4)

BENCHMARK 2
A. Solving Equations

6. Identify the Number of Solutions to an Equation

Vocabulary **Identity** An equation that is true for all values of the variable.

EXAMPLE **Solve the equation, if possible.**

a. $-8x = -4(2x + 1)$ **b.** $-5x - 15 = -5(x + 3)$

Solution:

An equation can have *one* solution, *no* solution, or *all real numbers* as solutions.

a.

$-8x = -4(2x + 1)$	**Write original equation.**
$-8x = -8x - 4$	**Distributive property**
$-8x + 8x = -8x - 4 + 8x$	**Add 8x to each side.**
$0 = -4$ ✗	**Simplify.**

The statement $0 = -4$ is not true, so the equation has no solution.

b.

$-5x - 15 = -5(x + 3)$	**Write original equation.**
$-5x - 15 = -5x - 15$	**Distributive property**

The statement $-5x - 15 = -5x - 15$ is true for all values of x. So the equation is an identity, and the solution is all real numbers.

PRACTICE **Solve the equation, if possible.**

16. $7(3s - 3) = 3(7s - 7)$ **17.** $-6 + 3(v - 9) = 6v + 27$

18. $6a + 1 = 2(3a - 1)$

Quiz

Use addition or subtraction to solve the equation.

1. $w + 3.6 = 8.9$ **2.** $p - 7.2 = -5$ **3.** $v + 12 = -3$

Use multiplication or division to solve the equation.

4. $5n = -6$ **5.** $\frac{g}{2.1} = 5$ **6.** $1.2z = 8.4$

Solve the equation.

7. $9x - 2 = 0$ **8.** $-4 + \frac{b}{2.5} = 40$

9. $24 = \frac{v}{6} - 3$ **10.** $9y + 5(y - 9) = 39$

11. $-c + 4(8 - c) = -43$ **12.** $\frac{1}{4}m - \left(\frac{3}{4}m + 2\right) = 8$

13. $\frac{1}{2}n + 4 = -\frac{3}{2}n - 18$ **14.** $4.3 + 2.3r = 7.1 - 1.9r$

15. $z - 26 = 5z - 36$

Solve the equation, if possible.

16. $4(b - 5) = 5(b - 4)$ **17.** $8p - 12 = -4(-2p + 3)$

18. $5(k + 3) - k = 3k + 5$

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BENCHMARK 2
(Chapters 3 and 4)

B. Proportion and Percent Problems (pp. 23–26)

The comparison of two quantities by division is called a *ratio*. The following examples illustrate how to write and use ratios.

1. Write a Ratio

The *ratio* of two quantities a and b can be written in three ways: a to b, $a : b$, or $\frac{a}{b}$.

EXAMPLE **Kim has a jar containing 45 pennies, 18 nickels, 30 dimes, and 42 quarters. Write the specified ratio in simplest form.**

a. number of nickels to number of pennies **b.** number of quarters to number of dimes **c.** number of pennies to total number of coins

Solution:

a. $\frac{\text{nickels}}{\text{pennies}} = \frac{18}{45} = \frac{2}{5}$

b. $\frac{\text{quarters}}{\text{dimes}} = \frac{42}{30} = \frac{7}{5}$

c. $\frac{\text{pennies}}{\text{total}} = \frac{45}{45 + 18 + 30 + 42} = \frac{45}{135} = \frac{1}{3}$

PRACTICE **On his last report card, Jay earned 2 A's, 3 B's, and 1 C. Write the specified ratio in simplest form.**

1. the number of A's to the number of B's
2. the number of C's to the number of A's and B's
3. the number of B's to the total number of grades

A school band orders t-shirts. They order 12 smalls, 10 mediums, and 15 larges. Write the specified ratio in simplest form.

4. the number of smalls to the total number of t-shirts
5. the number of mediums to the number of larges
6. the number of larges and smalls to the number of mediums

2. Solve a Proportion

Vocabulary **Proportion** An equation showing that two ratios are equivalent.

EXAMPLE **Solve the proportion $\frac{2}{3} = \frac{x}{15}$.**

Solution:

$\frac{2}{3} = \frac{x}{15}$ **Write original proportion.**

$15 \cdot \frac{2}{3} = 15 \cdot \frac{x}{15}$ **Multiply each side by 15.**

$\frac{30}{3} = x$ **Simplify.**

$10 = x$ **Divide.**

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BENCHMARK 2

(Chapters 3 and 4)

PRACTICE **Solve the proportion.**

7. $\frac{7}{42} = \frac{t}{84}$ **8.** $\frac{5}{6} = \frac{k}{72}$ **9.** $\frac{a}{65} = \frac{6}{39}$

10. $\frac{v}{3} = \frac{85}{51}$ **11.** $\frac{8}{20} = \frac{n}{15}$ **12.** $\frac{q}{54} = \frac{8}{36}$

3. Use the Cross Products Property

Vocabulary **Cross product** The product of the numerator of one ratio in a proportion and the denominator of the other ratio in the proportion.

EXAMPLE **Solve the proportion $\frac{2}{5} = \frac{x}{20}$.**

Solution:

Cross Products Property: The cross products of a proportion are equal.

$\frac{2}{5} = \frac{x}{20}$	Write original proportion.
$2 \cdot 20 = 5 \cdot x$	Cross products property
$40 = 5x$	Simplify.
$8 = x$	Divide each side by 5.

PRACTICE **Use the cross products property to solve the proportion.**

13. $\frac{y}{12} = \frac{19}{4}$ **14.** $\frac{15}{35} = \frac{p}{63}$ **15.** $\frac{13}{36} = \frac{65}{w}$

16. $\frac{7}{30} = \frac{196}{m-5}$ **17.** $\frac{5}{8} = \frac{n}{n+9}$ **18.** $\frac{18}{v-1} = \frac{48}{3v-5}$

4. Find a Percent Using a Proportion

EXAMPLE **What percent of 50 is 12?**

Solution:

Represent "*a* is *p* percent of *b*" using the proportion $\frac{a}{b} = \frac{p}{100}$.

$\frac{a}{b} = \frac{p}{100}$	Write proportion.
$\frac{12}{50} = \frac{p}{100}$	Substitute 12 for *a* and 50 for *b*.
$1200 = 50p$	Cross products property
$24 = p$	Divide each side by 50.

12 is 24% of 50.

PRACTICE **Use a proportion to answer the question.**

19. What is 34% of 60? **20.** 5 is 4% of what number?

21. 16 is what percent of 20? **22.** 9 is what percent of 50?

23. What is 5% of 76? **24.** 17 is 25% of what number?

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BENCHMARK 2
(Chapters 3 and 4)

C. Rewriting Equations in Two or More Variables (pp. 27–28)

A literal equation describes the relationship between two or more variables. The equation can be rewritten, or "solved," to isolate any one of the variables. The following examples show how to solve and use literal equations.

1. Solve a Literal Equation

Vocabulary **Literal equation** An equation, such as a formula, with two or more variables where the coefficients and constants have been replaced by letters.

EXAMPLE **Solve $y = mx + b$ for m.**

Remember that inverse operations apply to variables as well as to constants.

Solution:

$y = mx + b$	**Write original equation.**
$y - b = mx$	**Subtract b from each side.**
$\frac{y - b}{x} = m$	**Assume $x \neq 0$. Divide each side by x.**

PRACTICE **Solve the literal equation for the specified variable.**

1. $I = Prt$ for P **2.** $A = \pi r^2$ for r **3.** $F + V = E + 2$ for V

4. $V = \ell wh$ for w **5.** $V = \frac{1}{3}\pi r^2 h$ for h **6.** $A = \frac{1}{2}(b_1 + b_2)h$ for b_1

2. Use the Solution to a Literal Equation

EXAMPLE **Use the solution to the literal equation from the example in Part 1 to solve $14 = m \cdot 6 - 4$.**

After solving a literal equation, you can use unit analysis to check your work.

Solution:

$\frac{y - b}{x} = m$	**Solution of literal equation.**
$\frac{14 - (-4)}{6} = m$	**Substitute 14 for y, 6 for x, and -4 for b.**
$3 = m$	**Simplify.**

PRACTICE **Solve the given formula for the unknown variable. Then use the solution to answer the question.**

7. The density d of a substance is given by $d = \frac{m}{V}$, where m is the mass in grams (g) and V is volume in cubic centimeters (cm^3). A scientist completely fills a beaker with 28 g of a substance that has density 0.4375 g/cm^3. What is the beaker's volume?

8. The strength s of a radio signal is given by $s = \frac{1600}{d^2}$, where d is the distance in miles from the transmitter. If s is 100, how far are you from the transmitter?

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(Chapters 3 and 4)

9. A roller coaster car goes down a hill and then makes a loop. The velocity v of the car at the top of the loop is $v = 8\sqrt{h - 2r}$, where h is the hill's height and r is the loop's radius. If v is 32 ft/s and r is 15 ft, how tall is the hill?

3. Rewrite an Equation

EXAMPLE **Write $3x - 5y = 15$ so that y is a function of x.**

Remember to first isolate the *term* containing the dependent variable. Then multiply or divide to isolate the *variable*.

Solution:

$3x - 5y = 15$ **Write original equation.**

$-5y = 15 - 3x$ **Subtract $3x$ from each side.**

$y = -3 + \frac{3}{5}x$ **Divide each side by -5.**

PRACTICE **Write the equation so that y is a function of x.**

10. $2x - y = 10$ **11.** $8 + 3y = -4x$ **12.** $9y + 27 = -x$

13. $\frac{3}{4}y - 2x = 12$ **14.** $5x + \frac{2}{5}y = 30$ **15.** $-24 - 16y = 8x$

BENCHMARK 2 C. Equations in Two Variables

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D. Graphing Linear Equations (pp. 29–31)

The x-axis and y-axis divide a coordinate plane into four equal parts called **quadrants.** The quadrants are labeled with roman numerals I, II, III, and IV, moving counter-clockwise from the upper right quadrant. Each point in a coordinate plane has a unique ordered pair (x, y) that describes the point's location with respect to the origin $(0, 0)$.

The **solution** of an equation is the set of all ordered pairs (x, y) that make the equation a true statement. The graph of an equation is a graph of all the ordered pairs that make up the solution of the equation.

1. Plot Points in a Coordinate Plane

EXAMPLE **Plot each point and describe its location.**

a. $P(-4, -1)$ **b.** $Q(3, -4)$ **c.** $R(-2, 0)$ **d.** $S(0, -3)$

Solution:

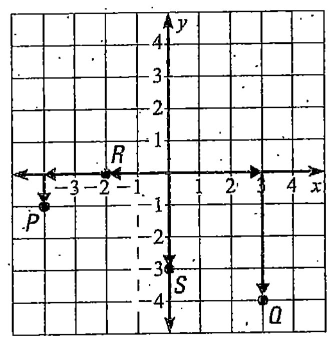
Another name for the x-coordinate is *abscissa*. Another name for the y-coordinate is *ordinate*.

a. Start at the origin. Move 4 units left, then 1 unit down. Point P is in Quadrant III.

b. Start at the origin. Move 3 units right, then 4 units down. Point Q is in Quadrant IV.

c. Start at the origin. Move 2 units left. Point R is on the x-axis.

d. Start at the origin. Move 3 units down. Point P is on the y-axis.



PRACTICE **Plot each point and describe its location.**

1. $A(3, 5)$ **2.** $B(-4, 0)$ **3.** $C(-1, 4)$

4. $D(0, -1)$ **5.** $E(-2, -3)$ **6.** $F(1, -4)$

2. Identify Solutions to Equations in Two Variables

EXAMPLE **Tell whether the ordered pair is a solution of the equation.**

a. $x + 2y = 8; (-4, 6)$ **b.** $5x - 2y = 10; (2, 1)$

Solution:

a. $x + 2y = 8$ **Write original equation.**

$(-4) + 2(6) \stackrel{?}{=} 8$ **Substitute −4 for x and 6 for y.**

$8 = 8$ ✓ **Simplify.**

$(-4, 6)$ is a solution.

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b. $5x - 2y = 10$ **Write original equation.**

$5(2) - 2(1) \stackrel{?}{=} 10$ **Substitute 2 for x and 1 for y.**

$8 \neq 10$ ✗ **Simplify.**

(2, 1) is not a solution.

PRACTICE **Tell whether the ordered pair is a solution of the equation.**

7. $-2x + 3y = 4; \left(0, \frac{4}{3}\right)$

8. $-8 = y; (-5, -8)$

9. $3x - 4y = -1; (-3, -4)$

10. $x = -2; (-1, -2)$

11. $y - 5x = -3; (-2, -13)$

12. $-4y + 2x = 0; \left(-\frac{1}{2}, \frac{1}{4}\right)$

3. Graph an Equation Using a Table

EXAMPLE **Graph the equation $-3x + y = 1$.**

Solution:

Step 1: **Solve** the equation for y:

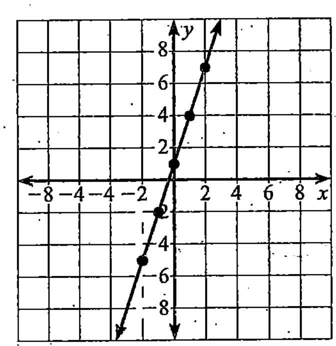
$$-3x + y = 1$$

$$y = 3x + 1$$

Step 2: **Make** a table by choosing a few values for x and finding the values of y.

You can choose any (x, y) pair from the graph and substitute it in the equation to make a true statement.

x	−2	−1	0	1	2
y	−5	−2	1	4	7



Step 3: **Plot** the points. Notice that the points appear to lie on a line.

Step 4: **Connect** the points by drawing a line through them. Use arrows to indicate that the graph goes on without end.

PRACTICE **Graph the equation.**

13. $x + y = 3$

14. $y - 2x = -1$

15. $-3x + 2y = 2$

16. $x - 3y = 3$

17. $4y - 3x = 8$

18. $2y - 5x = 0$

4. Graph Horizontal and Vertical Lines

Vocabulary

Linear equation An equation that can be written in the form $Ax + By = C$, where A, B, and C are real numbers and A and B are not both equal to zero. The graph of a linear equation is a straight line. When $A = 0$, the graph of the linear equation is a horizontal line. When $B = 0$, the graph of the linear equation is a vertical line.

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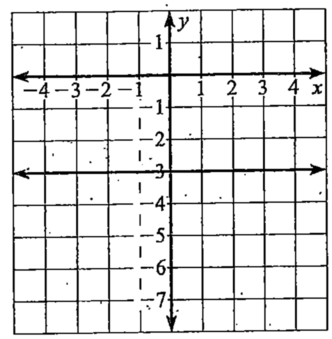
EXAMPLE **Graph the equation.**

a. $y = -3$ **b.** $x = 1$

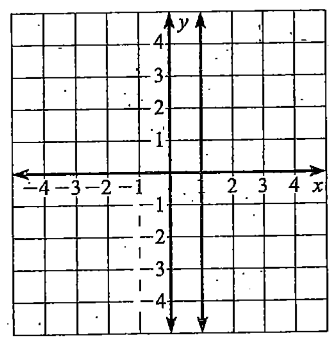
Solution:

All the solutions of $y = -3$ are ordered pairs in the form $(x, -3)$.

a. Notice that x can be any real number, but that y is always -3. The graph of the equation $y = -3$ is a horizontal line 3 units below the x-axis.



b. Notice that x will always be 1, but that y can be any real number. The graph of the equation $x = 1$ is a vertical line 1 unit to the right of the y-axis.



PRACTICE **Graph the equation.**

19. $x = -5$ **20.** $y = 1$ **21.** $2y = -3$

22. $2x - 1 = 0$ **23.** $x - 3 = 0$ **24.** $y + 2 = 0$

Quiz

Plot each point and describe its location.

1. $A(4, -7)$ **2.** $B(-9, -2)$ **3.** $C(0, 7)$

4. $D(1, 3)$ **5.** $E(-6, 0)$ **6.** $F(-4, 8)$

Tell whether the ordered pair is a solution of the equation.

7. $-5 = y; (5, -5)$ **8.** $-x + 4y = 4; (4, 5)$ **9.** $-8y + 4x = 0; \left(-2, -\frac{1}{4}\right)$

10. $y - 2x = -6; \left(\frac{1}{2}, -5\right)$ **11.** $x = -9; (1, -9)$ **12.** $3x - 7y = -4; (1, 1)$

Graph the equation.

13. $x - y = -2$ **14.** $y + 3x = -4$ **15.** $-5x + 3y = 2$

16. $y = -7$ **17.** $-3y - 2x = 9$ **18.** $x = 8$

19. $4y - 6x = 0$ **20.** $3 = -x$ **21.** $2y + 5 = -3$

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BENCHMARK 2
(Chapters 3 and 4)

E. Slope-Intercept Form and Direct Variation (pp. 32–35)

For any two points, there is one and only one line that contains both points. This fact can help you graph a linear equation. Many times, it will be convenient to use the points where the line crosses the x-axis and y-axis. These points are the **intercepts**. Knowing how steep the line is, or the **slope** of the line, also can help you graph a linear equation. If the graph of a linear equation passes through the origin (0, 0), the relationship between x and y is called a **direct variation**.

1. Find the Intercepts of the Graph of an Equation

Vocabulary

x-intercept The x-coordinate of the point where a graph intersects the x-axis.

y-intercept The y-coordinate of the point where a graph intersects the y-axis.

EXAMPLE **Find the x-intercept and the y-intercept of the graph of $3x + 4y = 12$.**

Solution:

To find the x-intercept, substitute 0 for y and solve for x.

$3x + 4y = 12$ **Write original equation.**

$3x + 4(0) = 12$ **Substitute 0 for y.**

$x = \frac{12}{3} = 4$ **Solve for x.**

Remember that the x- and y-intercepts are numbers, NOT ordered pairs.

To find the y-intercept, substitute 0 for x and solve for y.

$3x + 4y = 12$ **Write original equation.**

$3(0) + 4y = 12$ **Substitute 0 for x.**

$y = \frac{12}{4} = 3$ **Solve for y.**

The x-intercept is 4. The y-intercept is 3.

PRACTICE **Find the x-intercept and the y-intercept of the graph of the equation.**

1. $x + y = -6$ **2.** $-3y + 8 = -12x$ **3.** $4.5x + 0.5y = 9$

4. $-7y = 14x$ **5.** $-15 + 10y = 60x$ **6.** $3 - 18x = -6y$

2. Find the Slope of a Line

Vocabulary

Slope Describes how quickly a line rises or falls as it moves from left to right. Slope is the ratio m of the vertical change between two points on the line to the horizontal change between the same two points.

For points (x_1, y_1) and (x_2, y_2), $m = \frac{y_2 - y_1}{x_2 - x_1}$.

EXAMPLE **Find the slope of the line that passes through the points.**

a. (1, 5) and (4, 6) **b.** (−5, 7) and (3, −1)

c. (−2, 7) and (8, 7) **d.** (6, −8) and (6, 2)

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(Chapters 3 and 4)

Think of (x_1, y_1) as "the coordinates of the first point" and (x_2, y_2) as "the coordinates of the second point." Be sure to subtract the *x*- and *y*-coordinates in the same order.

Solution:

a. Let $(x_1, y_1) = (1, 5)$ and $(x_2, y_2) = (4, 6)$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ **Write formula for slope.**

$= \frac{6 - 5}{4 - 1} = \frac{1}{3}$ **Substitute and simplify.**

b. Let $(x_1, y_1) = (-5, 7)$ and $(x_2, y_2) = (3, -1)$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ **Write formula for slope.**

$= \frac{-1 - 7}{3 - (-5)} = \frac{-8}{8} = -1$ **Substitute and simplify.**

c. Let $(x_1, y_1) = (-2, 7)$ and $(x_2, y_2) = (8, 7)$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ **Write formula for slope.**

$= \frac{7 - 7}{8 - (-2)} = \frac{0}{10} = 0$ **Substitute and simplify.**

The slope is 0. The line is horizontal.

d. Let $(x_1, y_1) = (6, -8)$ and $(x_2, y_2) = (6, 2)$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ **Write formula for slope.**

$= \frac{2 - (-8)}{6 - 6} = \frac{10}{0}$ **Substitute. Division by 0 is undefined.**

The slope is undefined. The line is vertical.

PRACTICE **Find the slope of the line that passes through the points.**

7. $(6, -9)$ and $(-9, 6)$

8. $(4, 2)$ and $(4, 0)$

9. $(-11, 8)$ and $(13, 5)$

10. $(-1, -7)$ and $(1, -7)$

11. $(2.5, -5)$ and $(5.5, -9)$

12. $(-3, -5)$ and $(-2, 0)$

3. Graph an Equation Using Slope-Intercept Form

Vocabulary **Slope-intercept form** A linear equation in the form $y = mx + b$, where *m* is the slope and *b* is the *y*-intercept of the graph of the equation.

EXAMPLE **Graph the equation $-x + 2y = 4$.**

If you can substitute the coordinates of the second point in the original equation and get a true statement, then your graph is correct.

Solution:

Step 1: **Rewrite** the equation in slope-intercept form.

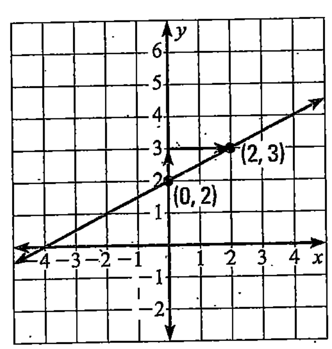
$y = \frac{1}{2}x + 2$

Step 2: **Identify** the slope and the *y*-intercept.

$m = \frac{1}{2}$ and $b = 2$.

Step 2: **Plot** the point that corresponds to the *y*-intercept, (0, 2).

Step 4: **Use** the slope to find another point on the line. Draw a line through the two points.



Name ______________________________ Date ________________

BENCHMARK 2

(Chapters 3 and 4)

PRACTICE **Graph the equation.**

13. $y = -\frac{2}{5}x + 7$ **14.** $-3x = 4y + 8$ **15.** $3x - 3y = 6$

16. $y = -4$ **17.** $-14x - 7y = 21$ **18.** $1.5y - 6x - 12 = 0$

4. Identify Direct Variation Equations

Vocabulary **Direct variation** An equation in the form $y = ax$, where $a \neq 0$, represents direct variation. The variable y *varies directly* with x.

Constant of variation The constant a in the direct variation equation $y = ax$.

EXAMPLE **Tell whether the equation represents direct variation. If so, identify the constant of variation.**

a. $6x - 4y = 0$ **b.** $x + y = 8$

Solution:

Try to rewrite the equation in the form $y = ax$.

a. $6x - 4y = 0$ **Write original equation.**

$-4y = -6x$ **Subtract $-6x$ from each side.**

$y = \frac{3}{2}x$ **Simplify.**

Because the equation $6x - 4y = 0$ can be rewritten in the form $y = ax$, it represents direct variation. The constant of variation is $\frac{3}{2}$.

b. $x + y = 8$ **Write original equation.**

$y = -x + 8$ **Subtract x from each side.**

Because the equation $x + y = 8$ cannot be rewritten in the form $y = ax$, it does not represent direct variation.

PRACTICE **Tell whether the equation represents direct variation. If so, identify the constant of variation.**

19. $y = -\frac{7}{8}x$ **20.** $x + 4 = 16y$ **21.** $9y = 5x$

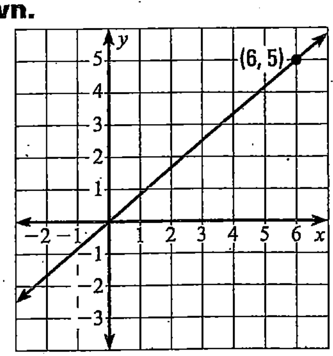
22. $x = -47y$ **23.** $-3 + x + 7 = -y + 4$ **24.** $13 = 26x$

5. Write and Use a Direct Variation Equation

EXAMPLE **The graph of a direct variation equation is shown.**

a. Write the direct variation equation.

b. Find the value of y when $x = 36$.



Name ______________________________ Date ______________

BENCHMARK 2

(Chapters 3 and 4)

Check the sign of the constant of variation in your equation. If the graph of $y = ax$ passes through Quadrants I and III, the constant should be positive. If the graph of $y = ax$ passes through Quadrants II and IV, the constant should be negative.

Solution:

a. Because y varies directly x, the equation has the form $y = ax$. Use the fact that $y = 5$ when $x = 6$ to find a.

$y = ax$ **Write direct variation equation.**

$5 = a(6)$ **Substitute.**

$\frac{5}{6} = a$ **Solve for *a*.**

A direct variation equation that relates x and y is $y = \frac{5}{6}x$.

b. When $x = 36$, $y = \frac{5}{6}(36) = 30$.

PRACTICE

Write the direct variation equation that passes through the given point. Then find the value of *y* for the given *x*.

25. $(3, -1); x = 12$ **26.** $(-4, -8); x = 32$ **27.** $(-6, 3); x = 18$

28. $(9, 2); x = 27$ **29.** $(-5, 7); x = 100$ **30.** $(-2, -1); x = 74$

Quiz

Find the *x*-intercept and the *y*-intercept of the graph of the equation.

1. $-21 + 14y = 84x$ **2.** $-3 + x = 3y$ **3.** $3.2x + 0.8y = 4$

Find the slope of the line that passes through the points.

4. $(8, -5)$ and $(-3, 4)$ **5.** $(1, 7)$ and $(-2, 7)$ **6.** $(-9, 7)$ and $(3, -5)$

Graph the equation.

7. $y = x + 1$ **8.** $y = -2$ **9.** $4x - 6y = 12$

Does the equation represent direct variation? If so, find the constant of variation.

10. $y = -\frac{4}{5}x$ **11.** $x + 3 = 9y$ **12.** $4y = 7x$

Write the direct variation equation that passes through the given point. Then find the value of *y* for the given *x*.

13. $(2, -5); x = 20$ **14.** $(-3, -9); x = 43$ **15.** $(-4, 6); x = 64$

Answers

Benchmark 2

A. Solving Equations in One Variable

1. -1 **2.** 11 **3.** 4 **4.** 20 **5.** 8 **6.** -27 **7.** 2
8. -3 **9.** -24 **10.** $-\frac{10}{7}$ **11.** 9 **12.** -5
13. 3 **14.** -6 **15.** $\frac{2}{13}$ **16.** All real numbers
17. -20 **18.** No solution

Quiz

1. 5.3 **2.** 2.2 **3.** -15 **4.** $-\frac{6}{5}$ **5.** 10.5 **6.** 7
7. $\frac{2}{9}$ **8.** 110 **9.** 162 **10.** 6 **11.** 15
12. -20 **13.** -11 **14.** $\frac{2}{3}$ **15.** $\frac{5}{2}$
16. 0 **17.** All real numbers **18.** -10

B. Proportion and Percent Problems

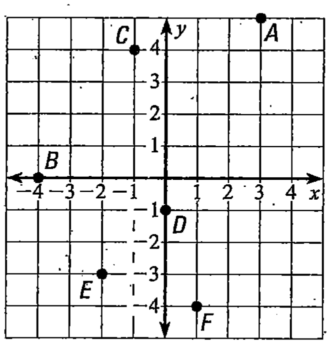
1. $\frac{2}{3}$ **2.** $\frac{1}{5}$ **3.** $\frac{1}{2}$ **4.** $\frac{12}{37}$ **5.** $\frac{2}{3}$ **6.** $\frac{27}{10}$ **7.** 14
8. 60 **9.** 10 **10.** 5 **11.** 6 **12.** 12 **13.** 57
14. 27 **15.** 180 **16.** 845 **17.** 15 **18.** 7
19. 20.4 **20.** 125 **21.** 80% **22.** 18% **23.** 3.8
24. 68

C. Rewriting Equations in Two or More Variables

1. $P = \frac{I}{rt}$ **2.** $r = \sqrt{\frac{A}{\pi}}$ **3.** $V = E - F + 2$
4. $w = \frac{V}{\ell h}$ **5.** $h = \frac{3V}{\pi r^2}$ **6.** $b_1 = \frac{2A}{h} - b_2$
7. $V = \frac{m}{d}$; 64 cm^3 **8.** $d = \frac{40}{\sqrt{s}}$; 4 miles
9. $h = \frac{v^2}{64} + 2r$; 46 ft **10.** $y = 2x - 10$
11. $y = -\frac{4}{3}x - \frac{8}{3}$ **12.** $y = -\frac{1}{9}x - 3$
13. $y = \frac{8}{3}x + 16$ **14.** $y = -\frac{25}{2}x + 75$
15. $y = -\frac{1}{2}x - \frac{3}{2}$

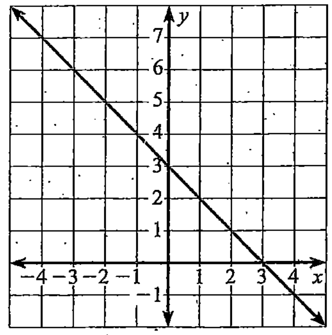
D. Graphing Linear Equations

1–6.



1. Quadrant I **2.** x-axis **3.** Quadrant II
4. y-axis **5.** Quadrant III **6.** Quadrant IV
7. Yes, it is a solution. **8.** Yes, it is a solution.
9. No, it is not a solution. **10.** No, it is not a solution. **11.** Yes, it is a solution. **12.** No, it is not a solution.

13.



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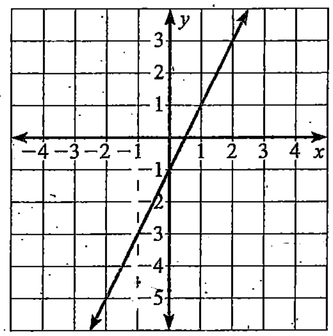

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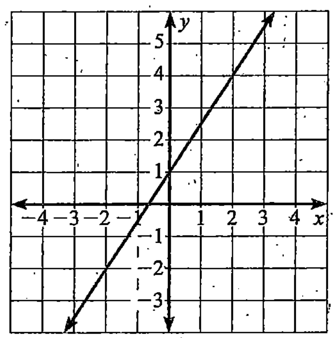

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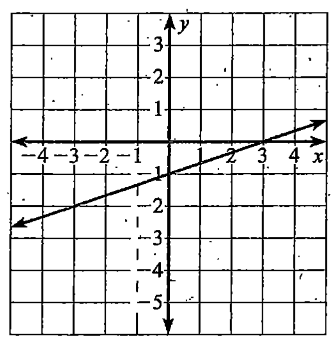

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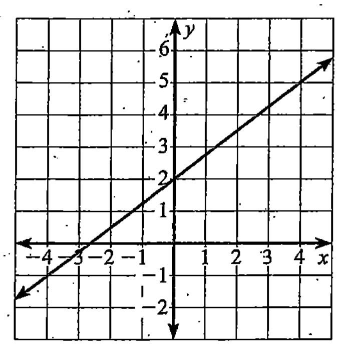
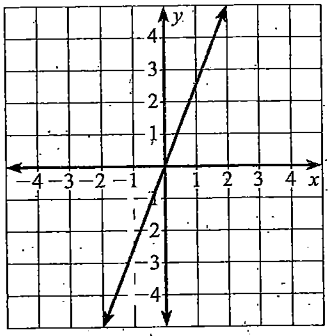


Answer Key

18.



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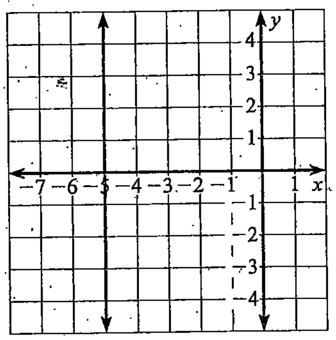

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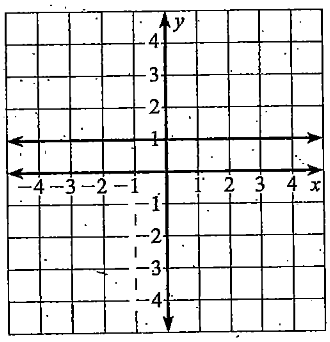

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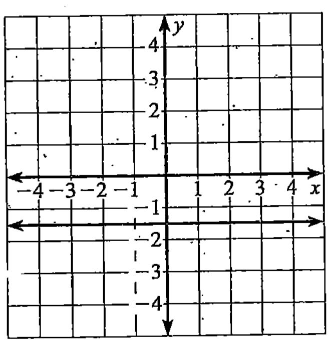

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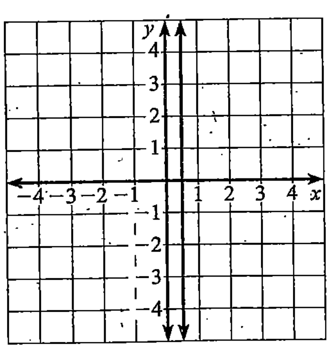

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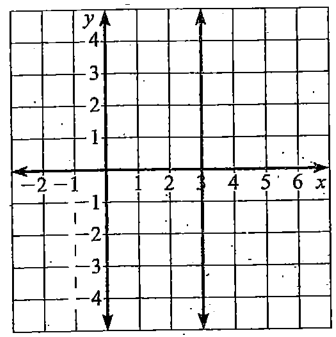

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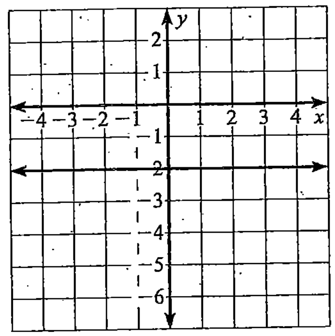
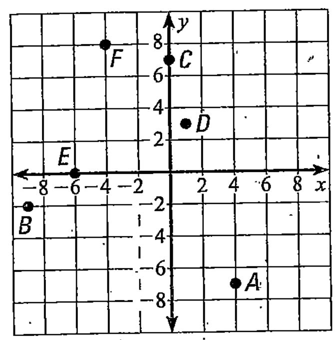


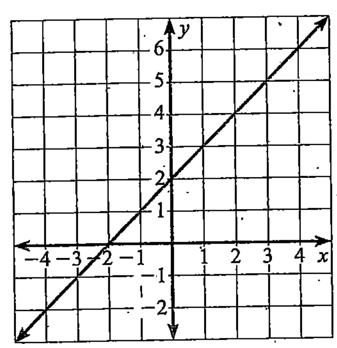
Quiz

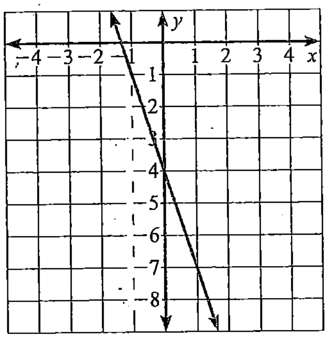
1–6.

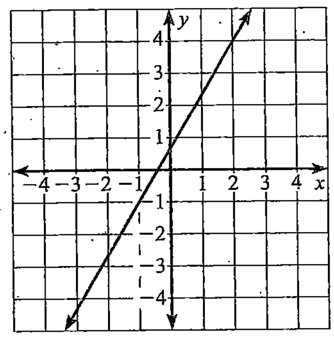


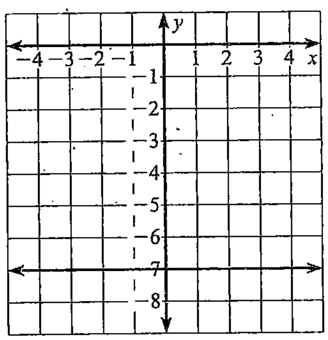
1. Quadrant IV **2.** Quadrant III **3.** y-axis **4.** Quadrant I **5.** x-axis **6.** Quadrant II **7.** Yes, it is a solution. **8.** No, it is not a solution. **9.** No, it is not a solution. **10.** Yes, it is a solution. **11.** No, it is not a solution. **12.** Yes, it is a solution.

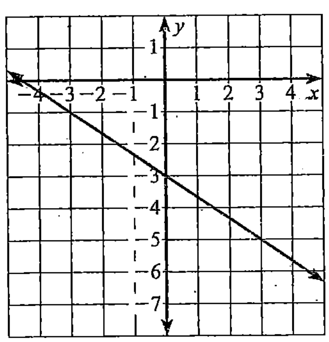
13.

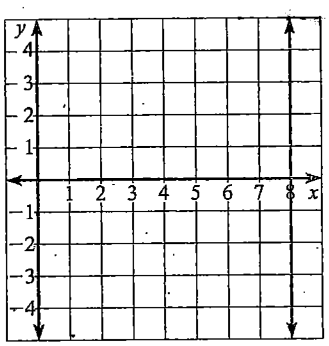


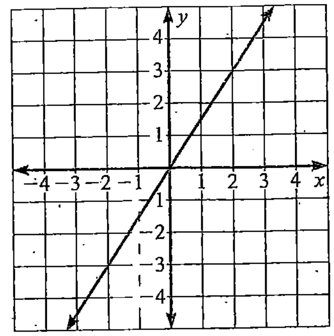
14. 

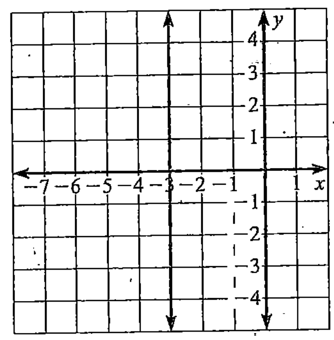
15. 

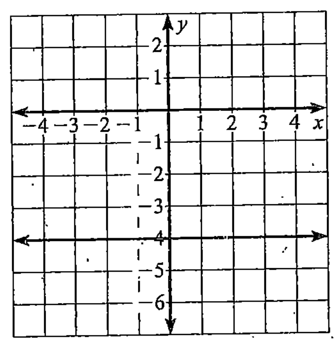
16. 

17. 

18. 

19. 

20. 

21. 

E. Slope-Intercept Form and Direct Variation

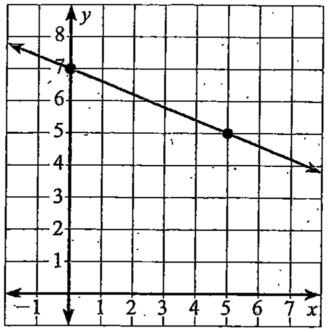
1. x-intercept: -6, y-intercept: -6

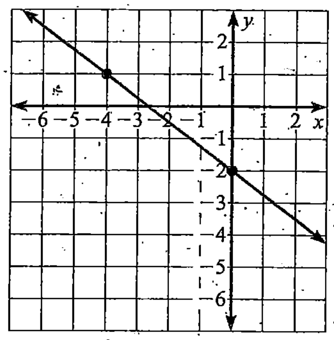
2. x-intercept: $-\frac{2}{3}$, y-intercept: $\frac{8}{3}$

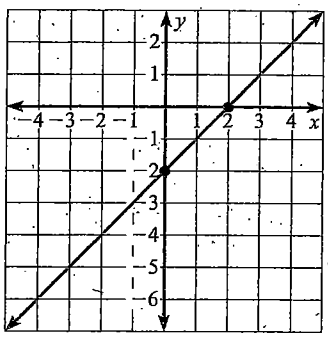
3. x-intercept: 2, y-intercept: 18 **4.** x-intercept: 0, y-intercept: 0 **5.** x-intercept: $-\frac{1}{4}$, y-intercept: $\frac{3}{2}$

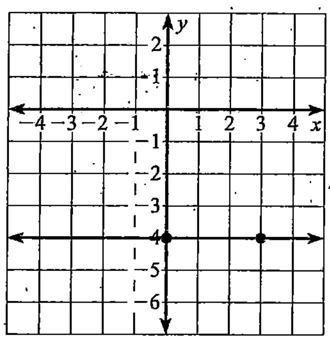
6. x-intercept: $\frac{1}{6}$, y-intercept: $-\frac{1}{2}$ **7.** -1

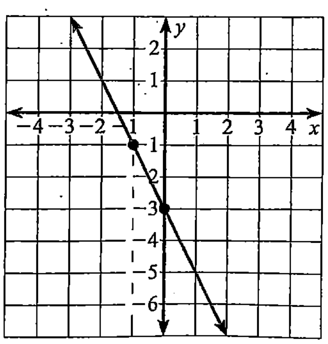
8. Undefined **9.** $-\frac{1}{8}$ **10.** 0 **11.** $-\frac{4}{3}$ **12.** 5

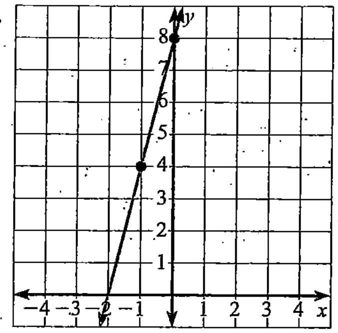
13. 

14. 

15. 

16. 

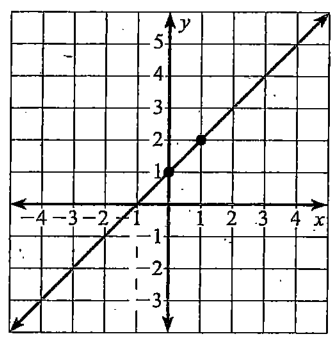
17. 

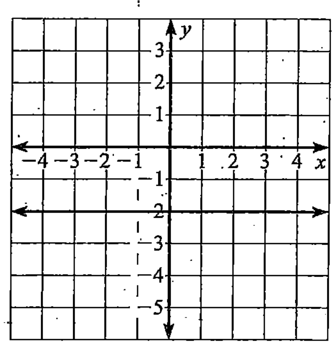
18. 

19. Yes; $-\frac{7}{8}$ **20.** No **21.** Yes; $\frac{5}{9}$ **22.** Yes; $-\frac{1}{47}$
23. Yes; -1 **24.** No **25.** $y = -\frac{1}{3}x$; -4
26. $y = 2x$; 64 **27.** $y = -\frac{1}{2}x$; -9
28. $y = \frac{2}{9}x$; 6 **29.** $y = -\frac{7}{5}x$; -140
30. $y = \frac{1}{2}x$; 37

Quiz

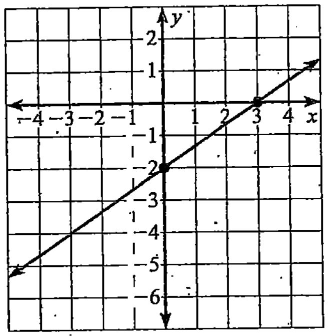
1. x-intercept: $-\frac{1}{4}$, y-intercept: $\frac{3}{2}$ **2.** x-intercept: 3, y-intercept: -1 **3.** x-intercept: 1.25, y-intercept: 5 **4.** $-\frac{9}{11}$ **5.** 0 **6.** -1

7. 

8. 

Answer Key

Answers *continued*

9. 

10. Yes; $-\frac{4}{5}$ **11.** No **12.** Yes; $\frac{7}{4}$

13. $y = -\frac{5}{2}x$; -50 **14.** $y = 3x$; 129

15. $y = -\frac{3}{2}x$; -96

Name ______________________ Date ____________

BENCHMARK 3
(Chapters 5 and 6)

A. Writing Linear Equations (pp. 36–40)

You can describe a line with equations in three different forms. You can write these equations if you know the slope and y-intercept of the line, if you know the slope and a point on the line, or if you know two points on the line. The following examples illustrate these three different forms of the equation of a line and show how to find them.

1. Write an Equation in Slope-Intercept Form

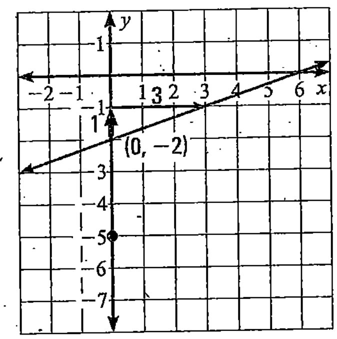
Vocabulary **Slope-intercept form** The equation $y = mx + b$, for a line with slope m and y-intercept b.

EXAMPLE **Write an equation of the line with a slope of $\frac{1}{3}$ and a y-intercept of -2.**

Solution:

$y = mx + b$ **Write slope-intercept form.**

$y = \frac{1}{3}x - 2$ **Substitute $\frac{1}{3}$ for m and -2 for b.**



PRACTICE **Write an equation of the line with the given slope and y-intercept.**

1. Slope is 6; y-intercept is -4.
2. Slope is -1; y-intercept is 3.
3. Slope is $\frac{3}{5}$; y-intercept is -5.
4. Slope is $\frac{2}{5}$; y-intercept is -3.
5. Slope is -4; y-intercept is 5.
6. Slope is $-\frac{1}{3}$; y-intercept is -2.

2. Write an Equation of a Line Given the Slope and a Point

EXAMPLE **Write an equation of the line that passes through $(4, -3)$ and has a slope of -2.**

Solution:

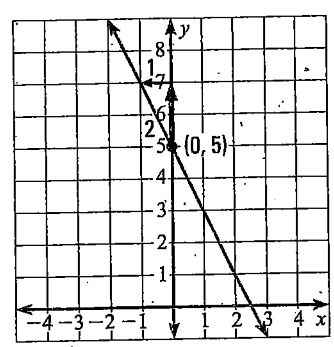
Step 1: Identify the slope. The slope is -2.

Step 2: Find the y-intercept. Substitute the slope and the coordinates of the given point in $y = mx + b$. Solve for b.

$y = mx + b$ **Write slope-intercept form.**

$-3 = -2(4) + b$ **Substitute -2 for m, 4 for x, and -3 for y.**

$5 = b$ **Solve for b.**



Make sure you don't switch the x and y values when you substitute.

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BENCHMARK 3

(Chapters 5 and 6)

Step 3: Write an equation of the line.

$y = mx + b$ **Write slope-intercept form.**

$y = -2x + 5$ **Substitute 2 for *m* and 5 for *b*.**

PRACTICE **Write an equation of the line that passes through the given point and has the given slope.**

7. $(-6, -2); m = \frac{4}{3}$

8. $(-1, 3); m = -\frac{1}{4}$

9. $(3, 4); m = -6$

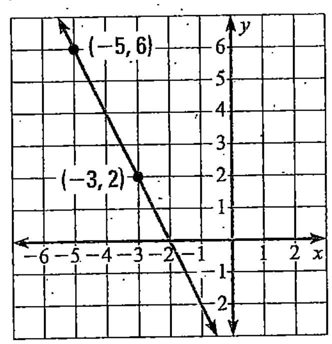
10. $(5, -3); m = \frac{3}{2}$

11. $(-3, 6); m = -\frac{2}{3}$

12. $(-1, -4); m = 2$

3. Write an Equation of a Line Given Two Points

EXAMPLE **Write an equation of the line shown.**



Solution:

Step 1: Calculate the slope using the formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 6}{-3 - (-5)} = \frac{-4}{2} = -2$$

Step 2: Find the y-intercept. Use the point $(-5, 6)$.

You also could find b by substituting the x and y values from the other known point, $(-3, 2)$.

$y = mx + b$ **Write slope-intercept form.**

$6 = -2(-5) + b$ **Substitute 6 for *y*, −2 for *m*, and −5 for *x*.**

$6 - 10 = b$ **Solve for *b*.**

$-4 = b$

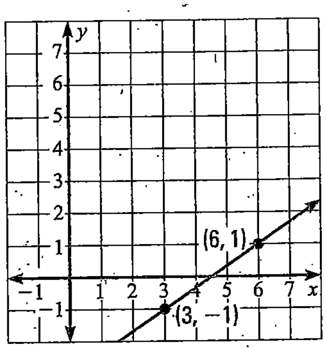
Step 3: Write an equation of the line.

$y = mx + b$ **Write slope-intercept form.**

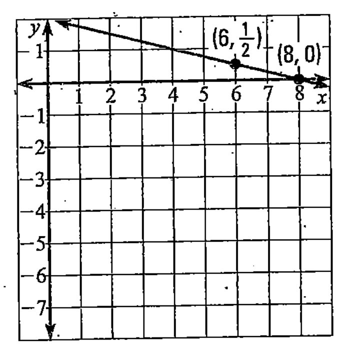
$y = -2x - 4$ **Substitute −2 for *m* and −4 for *b*.**

PRACTICE **Write an equation of the line shown.**

13.



14.



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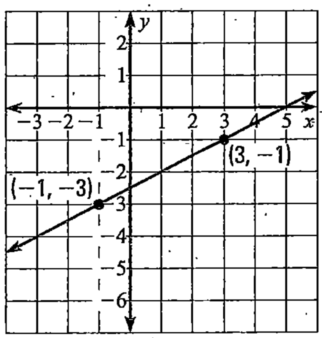
BENCHMARK 3

(Chapters 5 and 6)

15.



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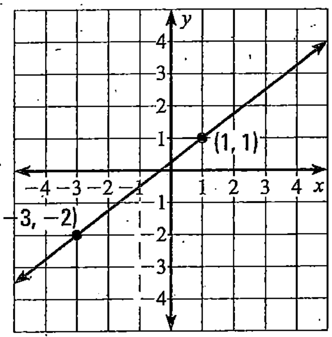

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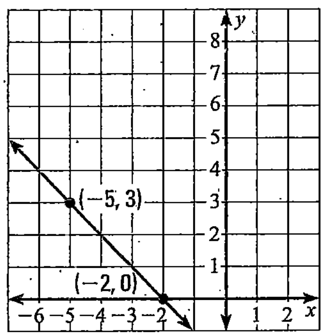

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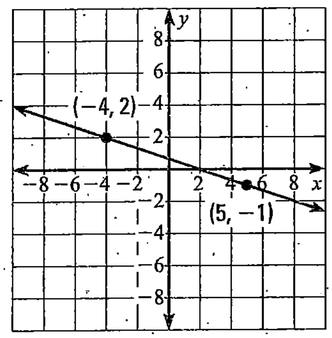


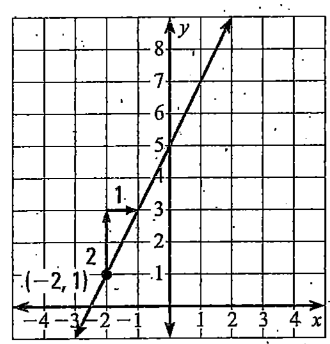
4. Write an Equation in Point-Slope Form

Vocabulary

Point-slope form The equation $y - y_1 = m(x - x_1)$, for the nonvertical line through a given point (x_1, y_1) with slope m.

EXAMPLE

Write an equation in point-slope form of the line that passes through the point (−2, 1) and has a slope of 2.



Solution:

Notice that (x_1, y_1) is a point of the line, and that m is the slope of the line.

$y - y_1 = m(x - x_1)$	**Write point-slope form.**
$y - 1 = 2(x + 2)$	**Substitute 1 for y_1, 2 for m, and −2 for x_1.**

PRACTICE

Write an equation in point-slope form of the line that passes through the given point and has the given slope.

19. $(3, -1);\ m = \frac{2}{3}$

20. $(4, 0);\ m = -\frac{1}{4}$

21. $(-3, -4);\ m = \frac{1}{2}$

22. $(1, 1);\ m = \frac{3}{4}$

23. $(-5, 3);\ m = -1$

24. $(-4, 2);\ m = -\frac{1}{3}$

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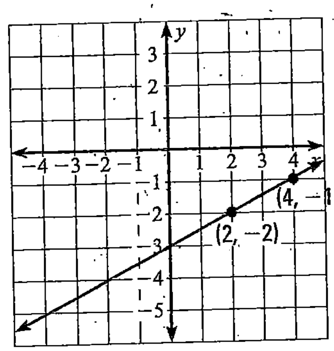
5. Write an Equation in Standard Form

Vocabulary **Standard form** The equation $Ax + By = C$, where A, B, and C are real numbers and A and B are not both zero.

EXAMPLE **Write an equation in standard form of the line shown.**



Solution:

Step 1: Calculate the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - (-2)}{4 - 2} = \frac{1}{2}$$

Step 2: Write an equation in point-slope form. Use $(2, -2)$.

$y - y_1 = m(x - x_1)$ **Write point-slope form.**

$y - (-2) = \frac{1}{2}(x - 2)$ **Substitute -2 for y_1, $\frac{1}{2}$ for m, and 2 for x_1.**

Step 3: Rewrite the equation in standard form.

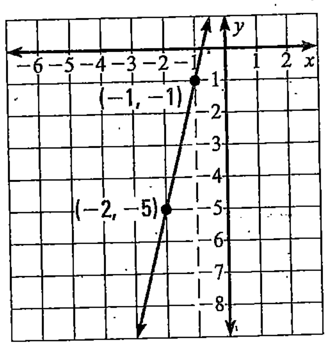
$y + 2 = \frac{1}{2}x - 1$ **Apply the distributive property.**

$2y + 4 = x - 2$ **Multiply each term by 2.**

$-x + 2y = -6$ **Simplify. Collect variable terms on one side, constants on the other.**

PRACTICE **Write an equation in standard form of the line shown.**

25.



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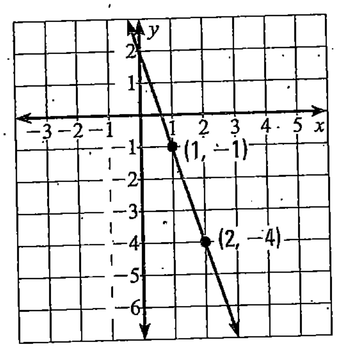

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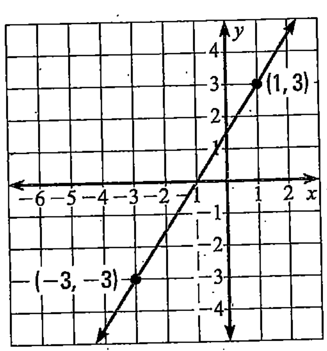

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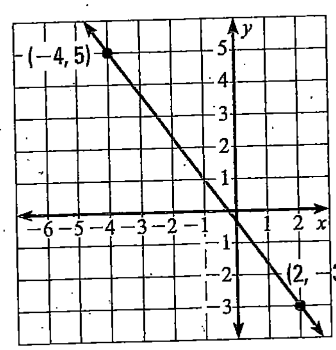

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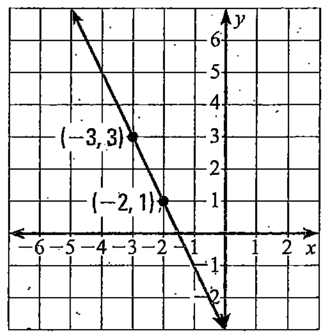
29.



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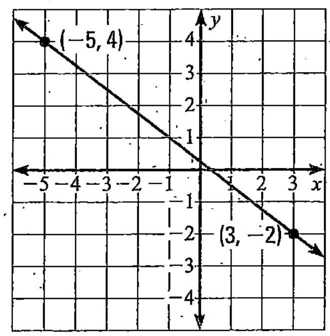


Quiz

Write an equation in slope-intercept form of the line with the given slope and y-intercept.

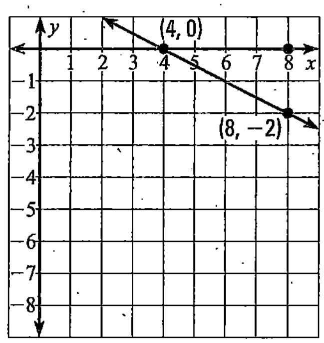
1. Slope is 4; y-intercept is 3.
2. Slope is -2; y-intercept is 1.
3. Slope is $\frac{5}{2}$; y-intercept is -4.
4. Slope is $-\frac{1}{3}$; y-intercept is -5.

Write an equation in the given form of the line that passes through the given point and has the given slope.

5. $(-3, -4)$; $m = \frac{1}{5}$ slope-intercept form
6. $(-2, 7)$; $m = -\frac{3}{4}$ point-slope form
7. $(1, 5)$; $m = -4$ point-slope form

Write equations in slope-intercept form and standard form of the line shown.

8.



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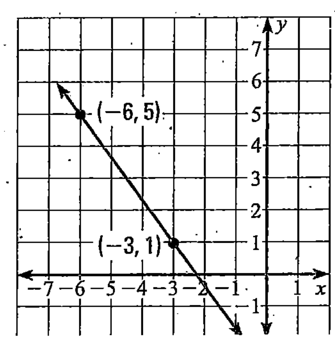

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BENCHMARK 3
(Chapters 5 and 6)

B. Parallel and Perpendicular Lines (pp. 41–43)

If two non-vertical lines in the same plane have the same slope, then they are parallel. If their slopes are negative reciprocals, then they are perpendicular. The converse is also true. If two non-vertical lines in the same plane are parallel, then they have the same slope. If they are perpendicular, then their slopes are negative reciprocals.

1. Determine Whether Lines are Parallel or Perpendicular

Vocabulary **Perpendicular lines** Lines in a plane that intersect to form a right (90°) angle.

EXAMPLE **Determine which lines, if any, are parallel or perpendicular.**

Line *a*: $y = 4x - 1$ **Line *b*:** $-4x + y = 3$ **Line *c*:** $2x + 8y = 4$

Solution:

Step 1: Write each equation in slope-intercept form. Find the slopes of the lines.

Line *a*: The equation is in slope-intercept form. The slope is 4.

Line *b*: $-4x + y = 3$

$y = 4x + 3$

Line *c*: $x + 4y = 4$

$y = -\frac{1}{4}x + 1$

The product of a non-zero slope m and its negative reciprocal is -1: $m\left(-\frac{1}{m}\right) = -1$

Step 2: **Compare** the slopes. Line a and line b have slopes of 4, so they are parallel. Line c has a slope of $-\frac{1}{4}$. $4\left(-\frac{1}{4}\right) = -1$, so it is perpendicular to lines a and b.

PRACTICE **Determine which lines, if any, are parallel or perpendicular.**

1. **Line *a*:** $y = \frac{3}{4}x + 2$ **Line *b*:** $4x - 3y = -3$ **Line *c*:** $3x - 4y = 20$
2. **Line *d*:** $x - 2y = 4$ **Line *e*:** $2x + y = 0$ **Line *f*:** $x + 2y = 3$
3. **Line *g*:** $5x + 7y = 7$ **Line *h*:** $y = \frac{7}{5}x + 3$ **Line *j*:** $7x - 5y = 2$

2. Write an Equation of a Parallel Line

EXAMPLE **Write an equation of the line that passes through (1, −2) and is parallel to the line $y = 5x + 2$.**

Solution:

Step 1: **Identify** the slope. The graph of the given equation has a slope of 5. So, the parallel line through (1, −2) will also have a slope of 5.

Step 2: **Find** the y-intercept. Use the slope and the given point.

$y = mx + b$	**Write slope-intercept form.**
$-2 = 5(1) + b$	**Substitute −2 for *y*, 5 for *m*, and 1 for *x*.**
$-7 = b$	**Solve for *b*.**

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BENCHMARK 3

(Chapters 5 and 6)

You can graph both lines to check your answer.

Step 3: Write an equation of the line in slope-intercept form.

$y = mx + b$ — Write slope-intercept form.

$y = 5x - 7$ — Substitute 5 for m and -7 for b.

PRACTICE **Write an equation of the line that passes through the given point and is parallel to the given line.**

4. $(-3, -1); y = \frac{4}{3}x + 1$
5. $(-8, 5); y = -\frac{1}{4}x - 2$
6. $(2, 3); y = -6x + 4$
7. $(2, 0); y = \frac{3}{2}x - 7$
8. $(-6, 4); y = -\frac{2}{3}x + 3$
9. $(-5, -2); y = 2x - 9$

3. Write an Equation of a Perpendicular Line

EXAMPLE **Write an equation of the line that passes through (4, 3) and is perpendicular to the line $y = 2x - 3$.**

Solution:

Step 1: Identify the slope. The graph of the given equation has a slope of 2. So, the slope of the perpendicular line through (4, 3) will be the negative reciprocal of 2, which is $-\frac{1}{2}$.

Step 2: Find the y-intercept. Use the slope and the given point.

$y = mx + b$ — Write slope-intercept form.

$3 = -\frac{1}{2}(4) + b$ — Substitute 3 for y, $-\frac{1}{2}$ for m, and 4 for x.

$5 = b$ — Solve for b.

Step 3: Write an equation of the line in slope-intercept form.

$y = mx + b$ — Write slope-intercept form.

$y = -\frac{1}{2}x + 5$ — Substitute $-\frac{1}{2}$ for m and 5 for b.

PRACTICE **Write an equation of the line that passes through the given point and is perpendicular to the given line.**

10. $(-3, -2); y = \frac{3}{2}x + 2$
11. $(-6, 1); y = -\frac{3}{4}x - 1$
12. $(2, 5); y = -8x + 3$
13. $(4, 0); y = \frac{1}{3}x - 4$
14. $(4, 6); y = -\frac{2}{3}x + 3$
15. $(-8, -2); y = 2x - 6$

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Quiz

Determine which lines, if any, are parallel or perpendicular.

1. Line *a*: $y = -\frac{3}{2}x + 4$ Line *b*: $3x + 2y = 2$ Line *c*: $2x - 3y = 3$
2. Line *d*: $x + 3y = 9$ Line *e*: $y = 3x - 2$ Line *f*: $3x + y = 2$
3. Line *g*: $x + 4y = 2$ Line *h*: $x - 4y = 0$ Line *j*: $y = \frac{1}{4}x + 1$

Write an equation of the line that passes through the given point and is parallel to the given line.

4. $(8, 1); y = \frac{3}{8}x$
5. $(-3, 3); y = -\frac{2}{3}x - 5$
6. $(-5, -2); y = 2x + 2$
7. $(-6, 2); y = \frac{4}{3}x + 4$
8. $(-8, 0); y = -\frac{1}{4}x - 3$
9. $(3, 2); y = -5x + 1$

Write an equation of the line that passes through the given point and is perpendicular to the given line.

10. $(1, 7); y = \frac{1}{3}x - 2$
11. $(6, 4); y = -\frac{2}{3}x + 6$
12. $(-4, -3); y = 2x - 7$
13. $(-6, 2); y = \frac{3}{2}x + 5$
14. $(3, -1); y = -\frac{3}{4}x - 8$
15. $(8, 2); y = -4x + 1$

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BENCHMARK 3
(Chapters 5 and 6)

C. Linear Models (pp. 44–48)

Paired data graphed in a **scatter plot** may show a **positive correlation**, a **negative correlation**, or no correlation. If there is a positive or negative correlation, the data can be modeled by a **line of fit** drawn close to the points on the scatter plot. The equation of this line will be in the form $y = mx + b$. Using **linear regression**, you can find the line that best fits the data. This **best-fitting line** or its equation can be used to approximate data points between or beyond known data points.

1. Describe the Correlation of Data

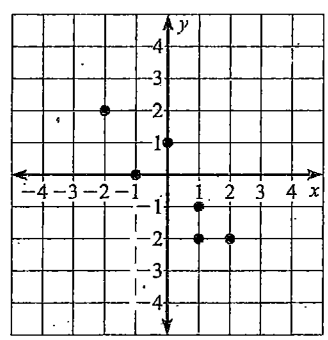
Vocabulary

Correlation The relationship between paired data; If the value of y tends to increase as the value of x increases, the correlation is positive. If the value of y tends to decrease as the value of x increases, the correlation is negative.

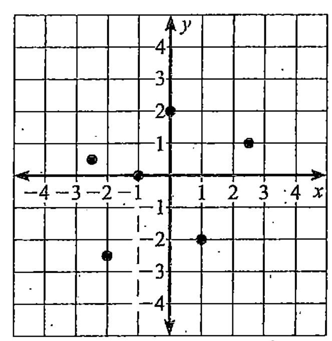
Scatter plot A graph that shows the relationship, if any, between paired data.

EXAMPLE **Describe the correlation, if any, of the data graphed in the scatter plot.**

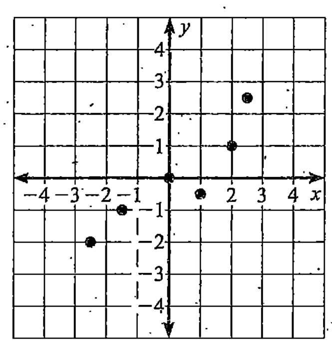
a.



b.



c.



Solution:

a. The value of y decreases as the value of x increases: *negative* correlation.

b. There is no apparent relationship between the value of y and x: *no* correlation.

c. The value of y increases as the value of x increases: *positive* correlation.

Benchmark 3

A. Writing Linear Equations

1. $y = 6x - 4$ **2.** $y = -x + 3$ **3.** $y = \frac{3}{5}x - 5$
4. $y = \frac{2}{5}x - 3$ **5.** $y = -4x + 5$
6. $y = -\frac{1}{3}x - 2$ **7.** $y = \frac{4}{3}x + 6$
8. $y = -\frac{1}{4}x + \frac{11}{4}$ **9.** $y = -6x + 22$
10. $y = \frac{3}{2}x - \frac{21}{2}$ **11.** $y = -\frac{2}{3}x + 4$
12. $y = 2x - 2$ **13.** $y = \frac{2}{3}x - 3$
14. $y = -\frac{1}{4}x + 2$ **15.** $y = \frac{1}{2}x - \frac{5}{2}$
16. $y = \frac{3}{4}x + \frac{1}{4}$ **17.** $y = -x - 2$
18. $y = -\frac{1}{3}x + \frac{2}{3}$ **19.** $y + 1 = \frac{2}{3}(x - 3)$
20. $y - 0 = -\frac{1}{4}(x - 4)$ **21.** $y + 4 = \frac{1}{2}(x + 3)$
22. $y - 1 = \frac{3}{4}(x - 1)$ **23.** $y - 3 = -(x + 5)$
24. $y - 2 = -\frac{1}{3}(x + 4)$ **25.** $-4x + y = 3$
26. $3x + y = 2$ **27.** $-3x + 2y = 3$
28. $4x + 3y = -1$ **29.** $2x + y = -3$
30. $3x + 4y = 1$

Quiz

1. $y = 4x + 3$ **2.** $y = -2x + 1$ **3.** $y = \frac{5}{2}x - 4$
4. $y = -\frac{1}{3}x - 5$ **5.** $y = \frac{1}{5}x - \frac{17}{5}$
6. $y - 7 = -\frac{3}{4}(x + 2)$ **7.** $y - 5 = -4(x - 1)$
8. $y = -\frac{1}{2}x + 2; x + 2y = 4$ **9.** $y = -\frac{4}{3}x - 3;$
$4x + 3y = -9$

B. Parallel and Perpendicular Lines

1. *a* and *c* are parallel. **2.** *d* and *e* are perpendicular. **3.** *h* and *j* are parallel. *g* is perpendicular to *h* and *j*. **4.** $y = \frac{4}{3}x + 3$
5. $y = -\frac{1}{4}x + 3$ **6.** $y = -6x + 15$
7. $y = \frac{3}{2}x - 3$ **8.** $y = -\frac{2}{3}x$ **9.** $y = 2x + 8$
10. $y = -\frac{2}{3}x - 4$ **11.** $y = \frac{4}{3}x + 9$
12. $y = \frac{1}{8}x + \frac{19}{4}$ **13.** $y = -3x + 12$
14. $y = \frac{3}{2}x$ **15.** $y = -\frac{1}{2}x - 6$

Quiz

1. *a* and *b* are parallel. *c* is perpendicular to *a* and *b*. **2.** *d* and *e* are perpendicular.
3. *h* and *j* are parallel. **4.** $y = \frac{3}{8}x - 2$
5. $y = -\frac{2}{3}x + 1$ **6.** $y = 2x + 8$
7. $y = \frac{4}{3}x + 10$ **8.** $y = -\frac{1}{4}x - 2$
9. $y = -5x + 17$ **10.** $y = -3x + 10$
11. $y = \frac{3}{2}x - 5$ **12.** $y = -\frac{1}{2}x - 5$
13. $y = -\frac{2}{3}x - 2$ **14.** $y = \frac{4}{3}x - 5$ **15.** $y = \frac{1}{4}x$

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BENCHMARK 5 Polynomial Operations

BENCHMARK 5

(Chapters 9 and 10)

A. Adding, Subtracting, and Multiplying Polynomials (pp. 80–83)

Real numbers can be represented by **polynomials**. Polynomials can be added, subtracted, and multiplied. The result of any of these operations on polynomials is another polynomial.

1. Identify and Classify Polynomials

Vocabulary

Monomial A number, a variable, or the product of a number and one or more variables with whole number exponents.

Degree of a monomial The greatest sum of the exponents of the variables in a monomial.

Polynomial A monomial or a sum of monomials, each called a *term*.

Degree of a polynomial The greatest sum of the exponents of the variables in a monomial.

Binomial A polynomial with two terms.

Trinomial A polynomial with three terms.

EXAMPLE **Tell whether the expression is a polynomial. If it is a polynomial, find its degree and classify it by the number of its terms. Otherwise, tell why it is not a polynomial.**

An exponent of a variable in a polynomial must be a whole number.

	Expression	Is it a polynomial?	Classify by degree and number of terms
a.	7	Yes	0 degree monomial
b.	$3x^3 + 2x$	Yes	3rd degree binomial
c.	$4d^x - d^2 + 3$	No; variable exponent	
d.	$2x^{-4} + 3x^2$	No; negative exponent	
e.	$6m^3n - 5mn^2 - 1$	Yes	4th degree trinomial

PRACTICE **Tell whether the expression is a polynomial. If it is a polynomial, find its degree and classify it by the number of its terms. Otherwise, tell why it is not a polynomial.**

1. $5x$ **2.** $4x^a - 3x + 2$ **3.** $2z^4 + 5z^2$

4. $2ab^4 + 4a^2b - 1$ **5.** $3y^{-3} - y^2 - 4$ **6.** $4m^3 - 5m^2 + m - 7$

2. Add and Subtract Polynomials

EXAMPLE **Find the sum.**

When a power of the variable appears in one polynomial but not the other, leave a space in that column, or write the term with a coefficient of 0 as a placeholder.

a. $(4x^3 - 3x^2 + 2) + (x^2 + 2x - 3)$ **b.** $(4x^2 + 3x - 4) + (x^2 - 2x + 1)$

Solution:

a. Vertical format: Align the terms in vertical columns.

$$\begin{array}{r} 4x^3 - 3x^2 + 2 \\ + x^2 + 2x - 3 \\ \hline 4x^3 - 2x^2 + 2x - 1 \end{array}$$

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BENCHMARK 5

(Chapters 9 and 10)

b. Horizontal format: Group like terms and simplify.

$$(4x^2 + 3x - 4) + (x^2 - 2x + 1) = (4x^2 + x^2) + [3x + (-2x)] + (-4 + 1)$$
$$= 5x^2 + x - 3$$

EXAMPLE **Find the difference.**

To subtract a polynomial, add its opposite. Write the subtraction as addition, and multiply *each* term in the polynomial by −1.

a. $(3x^2 + 2) - (-x^2 + 3x - 1)$ **b.** $(2x^2 - 4x + 4) - (x^2 - x + 1)$

Solution:

a.
$$\begin{array}{r} 3x^2 \qquad\quad + 2 \\ \underline{(-)\ -x^2 + 3x - 1} \end{array} \qquad \begin{array}{r} 3x^2 \qquad\quad + 2 \\ \underline{+\ \ x^2 - 3x + 1} \\ 4x^2 - 3x + 3 \end{array}$$

b. $(2x^2 - 4x + 4) - (x^2 - x + 1) = 2x^2 - 4x + 4 - x^2 + x - 1$
$= (2x^2 - x^2) + (-4x + x) + (4 - 1)$
$= x^2 - 3x + 3$

PRACTICE **Find the sum or difference.**

7. $(2x^2 + 3x - 1) + (x^3 - 4x + 3)$
8. $(x^2 + 2) - (4x^2 + 5x - 3)$
9. $(3x^4 - 3x^2 + 5) + (2x^3 - x^2 + 2x - 3)$
10. $(5x^3 + 2x^2 - x + 1) - (x^3 - x^2 - 2)$
11. $(4x^2 + x - 6) - (2x^2 - 3x + 2)$
12. $(4x^2 + 3x - 4) + (x^2 - 2x + 1)$

3. Multiply Polynomials

EXAMPLE **Find the product.**

a. $3x^2(2x^3 + x^2 - 3)$ **b.** $(2a^2 - a - 6)(5a + 2)$ **c.** $(y^2 - 3y + 4)(2y - 5)$

Solution:

a. $3x^2(2x^3 + x^2 - 3)$ — Write product.
$= 3x^2(2x^3) + 3x^2(x^2) - 3x^2(3)$ — Distributive property
$= 6x^5 + 3x^4 - 9x^2$ — Product of powers property

b. Step 1: Multiply by 2.

$$\begin{array}{r} 2a^2 - a - 6 \\ \underline{\times \qquad 5a + 2} \\ 4a^2 - 2a - 12 \end{array}$$

Align like terms vertically to help you add correctly.

Step 2: Multiply by $5a$.

$$\begin{array}{r} 2a^2 - \quad a - 6 \\ \underline{\times \qquad\quad 5a + 2} \\ 4a^2 - \ 2a - 12 \\ 10a^3 - 5a^2 - 30a \qquad\quad \end{array}$$

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BENCHMARK 5

(Chapters 9 and 10)

Step 3: Add products.

$$\begin{array}{r} 2a^2 - a - 6 \\ \times \quad 5a + 2 \\ \hline 4a^2 - 2a - 12 \\ 10a^3 - 5a^2 - 30a \quad \\ \hline 10a^3 - a^2 - 32a - 12 \end{array}$$

c. $(y^2 - 3y + 4)(2y - 5)$ — **Write product.**

$= y^2(2y - 5) - 3y(2y - 5) + 4(2y - 5)$ — **Distributive property**

$= 2y^3 - 5y^2 - 6y^2 + 15y + 8y - 20$ — **Distributive property**

$= 2y^3 - 11y^2 + 23y - 20$ — **Combine like terms.**

PRACTICE **Find the product.**

13. $4b^3(2b^2 + b - 3)$

14. $2x^4(2x^3 - 5x^2 - x + 6)$

15. $(y^2 - 3y + 5)(2y - 1)$

16. $(2z - 3)(z^2 + 4z - 1)$

17. $(a^2 - 5a - 2)(2a - 3)$

18. $(3x + 2)(2x^2 - 3x + 4)$

4. Find Special Products of Polynomials

EXAMPLE **Find the product.**

a. $(2x + 5)^2$ **b.** $(3y - 2)^2$ **c.** $(2x + y)(2x - y)$

Solution:

$(a + b)^2 = a^2 + 2ab + b^2$

a. $(2x + 5)^2 = (2x)^2 + 2(2x)(5) + (5)^2$ — **Square of a binomial pattern**

$= 4x^2 + 20x + 25$ — **Simplify.**

$(a - b)^2 = a^2 - 2ab + b^2$

b. $(3y - 2)^2 = (3y)^2 - 2(3y)(2) + (2)^2$ — **Square of a binomial pattern**

$= 9y^2 - 12y + 4$ — **Simplify.**

$(a + b)(a - b) = a^2 - b^2$

c. $(2x + y)(2x - y) = (2x)^2 - y^2$ — **Sum and difference pattern**

$= 4x^2 - y^2$ — **Simplify.**

PRACTICE **Find the product.**

19. $(2m + 5)^2$

20. $(z - 7)(z + 7)$

21. $(3x - 4)^2$

22. $(2x - 4)(2x + 4)$

23. $(2s + t)^2$

24. $(5x + 2)(5x - 2)$

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BENCHMARK 5

(Chapters 9 and 10)

Quiz

Tell whether the expression is a polynomial. If it is a polynomial, find its degree and classify it by the number of its terms. Otherwise, tell why it is not a polynomial.

1. $-6x^{-2}$

2. $3x + 1$

3. $x^3 + 2x^2 - 5x + 4$

4. $x^3 - 2x^2 + x^n - 7$

5. $3x^3y^2 + 2x^2y + xy$

6. $4x^2 - 3x + 1$

Evaluate the expression.

7. $(x^2 + 6x - 3) + (2x^3 - 2x + 5)$

8. $(5x^2 + 2x - 1) - (3x^2 - 4x + 2)$

9. $(x^2 + 3) - (-3x^2 - 2x + 6)$

10. $(2x^2 + x - 2) + (x^2 - 5x - 4)$

11. $3z^2(2z^3 - 4z + 5)$

12. $(5x + 1)(3x^2 - x - 7)$

13. $4b^3(b^3 - 2b^2 - 1)$

14. $(3y - 4)(3y + 4)$

15. $(k - 9)^2$

16. $(2x - 3)(x^2 + 5x - 2)$

17. $(2p + 3r)(2p - 3r)$

18. $(5x + 1)(2x - 3)$

19. $(3y + 5)^2$

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BENCHMARK 5
(Chapters 9 and 10)

B. Factoring Polynomials (pp. 84–87)

Polynomial equations can often be solved by **factoring**. To factor a polynomial, write it as a product of other polynomials. Then use the **zero-product property** to find the **roots** of the equation. The following examples describe methods that can be used to factor polynomial equations and find their solutions.

1. Use the Zero-Product Property

Vocabulary

Greatest common factor (GCF) of a polynomial A monomial with an integer coefficient that divides evenly into each term of the polynomial.

Zero-Product Property Let a and b be real numbers. If $ab = 0$, then $a = 0$ or $b = 0$.

Roots The solutions of an equation in which one side is zero and the other side is a product of polynomial factors.

EXAMPLE **Solve $(x - 3)(x + 1) = 0$.**

$(x - 3)(x + 1) = 0$ **Write original equation.**

$x - 3 = 0$ *or* $x + 1 = 0$ **Zero-product property**

$x = 3$ *or* $x = -2$ **Solve for x.**

PRACTICE **Use the zero-product property to solve the equation.**

1. $(x - 2)(x + 6) = 0$ **2.** $(x - 1)(x - 8) = 0$ **3.** $(x + 3)(x + 4) = 0$

2. Solve an Equation by Factoring

EXAMPLE **Solve the equation.**

a. $3x^2 - 6x = 0$ **b.** $5a^2 = 25a$

Solution:

a. $3x^2 - 6x = 0$ **Write original equation.**

$3x(x - 2) = 0$ **Factor left side.**

$3x = 0$ *or* $x - 2 = 0$ **Zero-product property**

$x = 0$ *or* $x = 2$ **Solve for x.**

b. $15a^2 = 20a$ **Write original equation.**

$15a^2 - 20a = 0$ **Subtract $20a$ from each side.**

$5a(3a - 4) = 0$ **Factor left side.**

$5a = 0$ *or* $3a - 4 = 0$ **Zero-product property**

$a = 0$ *or* $a = \frac{4}{3}$ **Solve for a.**

Write the equation so that one side is 0 in order to use the zero-product property.

PRACTICE **Solve the equation by factoring.**

4. $4b^2 - 16b = 0$ **5.** $x^2 + 7x = 0$ **6.** $8m^2 = -6m$

7. $5z^2 = 7z$ **8.** $6a^2 - 3a = 0$ **9.** $10x^2 = 2x$

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BENCHMARK 5

(Chapters 9 and 10)

★ Remember!
X box
Super Secret

3. Factor $x^2 + bx + c$

EXAMPLE **Factor the polynomial.**

$x^2 + bx + c = (x + p)(x + q)$ if $p + q = b$ and $pq = c$.

a. $x^2 + 8x + 12$ **b.** $t^2 - 7t + 10$ **c.** $z^2 - 5z - 14$

Solution:

To find two positive factors of 12 whose sum is 8, make an organized list.

a.

Factors of 12	Sum of factors	
12, 1	$12 + 1 = 13$	✗
6, 2	$6 + 2 = 8$	← Correct sum
4, 3	$4 + 3 = 7$	✗

The factors 6 and 2 have a sum of 8, so they are the correct values of p and q.

$x^2 + 8x + 12 = (x + 6)(x + 2)$

Both p and q must be negative, since b is negative and c is positive.

b.

Factors of 10	Sum of factors	
$-10, -1$	$-10 + (-1) = -11$	✗
$-5, -2$	$-5 + (-2) = -7$	← Correct sum

$t^2 - 7t + 10 = (x - 5)(x - 2)$

Since c is negative, p and q must have different signs.

c.

Factors of -14	Sum of factors	
$-14, 1$	$-14 + 1 = -13$	✗
$14, -1$	$14 + (-1) = 13$	✗
$-7, 2$	$-7 + 2 = -5$	← Correct sum
$7, -2$	$7 + (-2) = 5$	✗

$z^2 - 5z - 14 = (x - 7)(x + 2)$

PRACTICE **Factor the polynomial.**

10. $z^2 + 7z + 12$ **11.** $n^2 - 8n + 7$ **12.** $m^2 - 2m - 24$

13. $y^2 - 5y + 6$ **14.** $t^2 + 2t - 15$ **15.** $x^2 + 6x + 5$

4. Factor $ax^2 + bx + c$

EXAMPLE **Factor the polynomial.**

a. $3x^2 - 5x + 2$ **b.** $2t^2 + t - 3$ **c.** $-5m^2 + 9m + 2$

Solution:

When a is positive, consider the signs of b and c. Since b is negative and c is positive, both factors must be negative.

a.

Factors of 3	Factors of 2	Possible factorization	Middle term when multiplied	
1, 3	$-1, -2$	$(x - 1)(3x - 2)$	$-2x - 3x = -5x$	← Correct
1, 3	$-2, -1$	$(x - 2)(3x - 1)$	$-x - 6x = -7x$	✗

$3x^2 - 5x + 2 = (x - 1)(3x - 2)$

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BENCHMARK 5

(Chapters 9 and 10)

Since *b* is positive and *c* is negative, the factors of *c* must have different signs.

b.

Factors of 2	Factors of −3	Possible factorization	Middle term when multiplied	
1, 2	1, −3	$(t+1)(2t-3)$	$-3t+2t=-t$	✗
1, 2	−1, 3	$(t-1)(2t+3)$	$3t-2t=t$	← Correct
1, 2	3, −1	$(t+3)(2t-1)$	$-t+6t=5t$	✗
1, 2	−3, 1	$(t-3)(2t+1)$	$t-6t=-5t$	✗

$2t^2+t-3=(t-1)(2t+3)$

When *a* is negative, first factor −1 from each term of the trinomial. Then factor as in the previous examples.

Remember to include the −1 that you factored out earlier.

c. $-5m^2+9m+2=-(5m^2-9m-2)$

Factors of 5	Factors of −2	Possible factorization	Middle term when multiplied	
1, 5	1, −2	$(m+1)(5m-2)$	$-2m+5m=3m$	✗
1, 5	2, −1	$(m+2)(5m-1)$	$-m+10m=9m$	✗
1, 5	−1, 2	$(m-1)(5m+2)$	$2m-5m=-3m$	✗
1, 5	−2, 1	$(m-2)(5m+1)$	$m-10m=-9m$	← Correct

$-5m^2+9m+2=-(m-2)(5m+1)$

PRACTICE **Factor the polynomial.**

16. $4x^2-8x-5$ **17.** $2y^2-11y+9$ **18.** $-4z^2+4z+15$

19. $3m^2-10m+8$ **20.** $-2x^2+17x-21$ **21.** $3t^2+5t-12$

5. Factor Special Products

EXAMPLE **Factor the polynomial.**

a. $4x^2-25=0$ **b.** $4m^2-12m+9$ **c.** $9z^2+30z+25$

Difference of two squares: $a^2-b^2=(a+b)(a-b)$

Solution:

a. $4x^2-25=(2x)^2-5^2$ Write as a^2-b^2.

$=(2x+5)(2x-5)$ Difference of two squares pattern

Perfect square trinomial: $a^2+2ab+b^2=(a+b)^2$, $a^2-2ab+b^2=(a-b)^2$

b. $4m^2-12m+9=(2m)^2-2(2m\cdot 3)+3^2$ Write as $a^2-2ab+b^2$.

$=(2m-3)^2$ Perfect square trinomial pattern

c. $9z^2+30z+25=(3z)^2+2(3z\cdot 5)+5^2$ Write as $a^2+2ab+b^2$.

$=(3z+5)^2$ Perfect square trinomial pattern

PRACTICE **Factor the polynomial.**

22. $9x^2-1$ **23.** $4s^2+4s+1$ **24.** $4m^2-12mn+9n^2$

25. $16t^2-9$ **26.** $25z^2-20z+4$ **27.** $x^2+8xy+16y^2$

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BENCHMARK 5
(Chapters 9 and 10)

BENCHMARK 5 B. Factoring Polynomials

6. Factor by Grouping

Vocabulary

Factor by grouping In a polynomial of four terms, factor a common monomial from pairs of terms, then look for a common binomial factor.

EXAMPLE

Check your factorization by multiplying the factors. Or graph the original polynomial and the factors on a graphing calculator. The two graphs should coincide.

Factor the polynomial.

a. $m^3 + 4m^2 - 3m - 12$ **b.** $s^2 + 2st - 2s - 4t$

Solution:

a. $m^3 + 4m^2 - 3m - 12 = (m^3 + 4m^2) + (-3m - 12)$ **Group terms.**

$= m^2(m + 4) + (-3)(m + 4)$ **Factor each group.**

$= (m^2 - 3)(m + 4)$ **Distributive property**

b. $s^2 + 2st - 2s - 4t = (s^2 + 2st) + (-2s - 4t)$ **Group terms.**

$= s(s + 2t) + (-2)(s + 2t)$ **Factor each group.**

$= (s - 2)(s + 2t)$ **Distributive property**

PRACTICE

Factor the polynomial.

28. $2t^3 + 3t^2 - 10t - 15$ **29.** $2m^2 - 10m + mn - 5n$

30. $3x^3 - 21x^2 + x - 7$ **31.** $x^2 - 4x + 2xy - 8y$

Quiz

Factor the polynomial.

1. $4y^3 - 20y$ **2.** $10m^4 + 15m^2$ **3.** $t^2 - 7t - 18$

4. $x^2 + 6x - 27$ **5.** $4s^2 - 27s - 7$ **6.** $10y^2 + 9y + 2$

7. $16x^4 - y^2$ **8.** $25s^2 - 4t^2$ **9.** $m^2 - 6mn + 9n^2$

10. $4x^2 + 24xy + 36y^2$ **11.** $x^2 + 4xy - 3x - 12y$ **12.** $t^2 + 3st - 2t - 6s$

Factor and use the zero-product property to solve the equation.

13. $2x^2 + 6x = 0$ **14.** $r^2 + r - 20 = 0$ **15.** $y^2 - 6y = 16$

16. $3x^2 = 5 - 14x$ **17.** $4m^2 - 8m + 3 = 0$ **18.** $z^2 = 64$

19. $9n^2 - 4 = 0$ **20.** $9x^2 + 30x + 25 = 0$ **21.** $4z^2 + 1 = 4z$

Benchmark 5

A. Adding, Subtracting, and Multiplying Polynomials

1. Yes; 1st degree monomial **2.** No; variable exponent **3.** Yes; 4th degree binomial **4.** Yes; 5th degree trinomial **5.** No; negative exponent **6.** Yes; 3rd degree polynomial **7.** $x^3 + 2x^2 - x + 2$ **8.** $-3x^2 - 5x + 5$ **9.** $3x^4 + 2x^3 - 4x^2 + 2x + 2$ **10.** $4x^3 + 3x^2 - x + 3$ **11.** $2x^2 + 4x - 8$ **12.** $5x^2 + x - 3$ **13.** $8b^5 + 4b^4 - 12b^3$ **14.** $4x^7 - 10x^6 - 2x^5 + 12x^4$ **15.** $2y^3 - 7y^2 + 13y - 5$ **16.** $2z^3 + 5z^2 - 14z + 3$ **17.** $2a^3 - 13a^2 + 11a + 6$ **18.** $6x^3 - 5x^2 + 6x + 8$ **19.** $4m^2 + 20m + 25$ **20.** $z^2 - 49$ **21.** $9x^2 - 24x + 16$ **22.** $4x^2 - 16$ **23.** $4s^2 + 4st + t^2$ **24.** $25x^2 - 4$

Quiz

1. Not a polynomial; negative exponent **2.** Yes; 1st degree binomial **3.** Yes; 3rd degree polynomial **4.** No; variable exponent **5.** Yes; 5th degree trinomial **6.** Yes; 2nd degree trinomial **7.** $2x^3 + x^2 + 4x + 2$ **8.** $2x^2 + 6x - 3$ **9.** $4x^2 + 2x - 3$ **10.** $3x^2 - 4x - 6$ **11.** $6z^5 - 12z^3 + 15z^2$ **12.** $15x^3 - 2x^2 - 36x - 7$ **13.** $4b^6 - 8b^5 - 4b^3$ **14.** $9y^2 - 16$ **15.** $k^2 - 18k + 81$ **16.** $2x^3 + 7x^2 - 19x + 6$ **17.** $4p^2 - 9r^2$ **18.** $10x^2 - 13x - 3$ **19.** $9y^2 + 30y + 25$

B. Factoring Polynomials

1. $-6, 2$ **2.** $1, 8$ **3.** $-4, -3$ **4.** $0, 4$ **5.** $-7, 0$ **6.** $-\frac{3}{4}, 0$ **7.** $0, \frac{7}{5}$ **8.** $0, \frac{1}{2}$ **9.** $0, \frac{1}{5}$ **10.** $(z + 4)(z + 3)$ **11.** $(n - 7)(n - 1)$ **12.** $(m + 4)(m - 6)$ **13.** $(y - 2)(y - 3)$ **14.** $(t - 3)(t + 5)$ **15.** $(x + 5)(x + 1)$ **16.** $(2x - 5)(2x + 1)$ **17.** $(2y - 9)(y - 1)$ **18.** $-(2z + 3)(2z - 5)$ **19.** $(3m - 4)(m - 2)$ **20.** $-(2x - 3)(x - 7)$ **21.** $(3t - 4)(t + 3)$ **22.** $(3x + 1)(3x - 1)$ **23.** $(2s + 1)^2$ **24.** $(2m - 3n)^2$ **25.** $(4t + 3)(4t - 3)$ **26.** $(5z - 2)^2$ **27.** $(x + 4y)^2$ **28.** $(2t + 3)(t^2 - 5)$ **29.** $(m - 5)(2m + n)$ **30.** $(3x^2 + 1)(x - 7)$ **31.** $(x + 2y)(x - 4)$

Quiz

1. $4y(y^2 - 5)$ **2.** $5m^2(2m^2 + 3)$ **3.** $(t - 9)(t + 2)$ **4.** $(x - 3)(x + 9)$ **5.** $(s - 7)(4s + 1)$ **6.** $(5y + 2)(2y + 1)$ **7.** $(4x^2 + y)(4x^2 - y)$ **8.** $(5s + 2t)(5s - 2t)$ **9.** $(m - 3n)^2$ **10.** $(2x + 6y)^2$ **11.** $(x - 3)(x + 4y)$ **12.** $(t - 2)(t + 3s)$ **13.** $-3, 0$ **14.** $-5, 4$ **15.** $-2, 8$ **16.** $-5, \frac{1}{3}$ **17.** $\frac{1}{2}, \frac{3}{2}$ **18.** $-8, 8$ **19.** $-\frac{2}{3}, \frac{2}{3}$ **20.** $-\frac{5}{3}$ **21.** $\frac{1}{2}$

Answer Key

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BENCHMARK 6

(Chapters 11, 12, and 13)

Rationalizing the denominator the process of eliminating a radical from an expression's denominator

EXAMPLE **Use the product property of radicals.**

Remember that variables with even exponents are perfect squares.

a. $\sqrt{75} = \sqrt{25 \cdot 3}$ Factor using perfect square factor.

$= \sqrt{25} \cdot \sqrt{3}$ Product property of radicals

$= 5\sqrt{3}$ Simplify.

b. $\sqrt{4y^5} = \sqrt{4 \cdot y^4 \cdot y}$ Factor using perfect square factors.

$= \sqrt{4} \cdot \sqrt{y^4} \cdot \sqrt{y}$ Product property of radicals

$= 2y^2\sqrt{y}$ Simplify.

EXAMPLE **Use the quotient property of radicals.**

a. $\sqrt{\frac{5}{49}} = \frac{\sqrt{5}}{\sqrt{49}}$ Quotient property of radicals

$= \frac{\sqrt{5}}{7}$ Simplify.

b. $\sqrt{\frac{3}{z^2}} = \frac{\sqrt{3}}{\sqrt{z^2}}$ Quotient property of radicals

$= \frac{\sqrt{3}}{z}$ Simplify.

EXAMPLE **Rationalize the denominator.**

Multiply both the numerator and the denominator of the radical expression by the denominator of the radical expression.

a. $\frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$ Multiply by $\frac{\sqrt{5}}{\sqrt{5}}$.

$= \frac{3\sqrt{5}}{\sqrt{25}}$ Product property of radicals

$= \frac{3\sqrt{5}}{5}$ Simplify.

b. $\frac{\sqrt{5}}{\sqrt{2m}} = \frac{\sqrt{5}}{\sqrt{2m}} \cdot \frac{\sqrt{2m}}{\sqrt{2m}}$ Multiply by $\frac{\sqrt{2m}}{\sqrt{2m}}$.

$= \frac{\sqrt{10m}}{\sqrt{4m^2}}$ Product property of radicals

$= \frac{\sqrt{10m}}{\sqrt{4} \cdot \sqrt{m^2}}$ Product property of radicals

$= \frac{\sqrt{10m}}{2m}$ Simplify.

PRACTICE **Simplify the radical expression.**

5. $\sqrt{48}$ 6. $\sqrt{\frac{11}{25}}$ 7. $\frac{\sqrt{6x}}{\sqrt{5}}$ 8. $\sqrt{\frac{15}{x^4}}$

9. $\sqrt{\frac{12x}{z^2}}$ 10. $\frac{\sqrt{10}}{\sqrt{3a}}$ 11. $\sqrt{63}$ 12. $\sqrt{16y^3}$

13. $\sqrt{28yz^2}$ 14. $\frac{4}{\sqrt{3x}}$ 15. $\sqrt{\frac{20}{9x^2}}$ 16. $\sqrt{\frac{25}{4b^3}}$

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BENCHMARK 6
(Chapters 11, 12, and 13)

3. Add, Subtract and Multiply Radical Expressions

EXAMPLE **Simplify the radical expression.**

You can combine expressions only if they have the same radicand.

a. $6\sqrt{6} + \sqrt{7} - 2\sqrt{6} = 6\sqrt{6} - 2\sqrt{6} + \sqrt{7}$ — **Commutative property**

$= (6 - 2)\sqrt{6} + \sqrt{7}$ — **Distributive property**

$= 4\sqrt{6} + \sqrt{7}$ — **Simplify.**

b. $7\sqrt{2} + \sqrt{18} = 7\sqrt{2} + \sqrt{9 \cdot 2}$ — **Factor using perfect square factor.**

$= 7\sqrt{2} + \sqrt{9} \cdot \sqrt{2}$ — **Product property of radicals**

$= 7\sqrt{2} + 3\sqrt{2}$ — **Simplify.**

$= (7 + 3)\sqrt{2}$ — **Distributive property**

$= 10\sqrt{2}$ — **Simplify.**

EXAMPLE **Simplify the radical expression.**

a. $\sqrt{2}(\sqrt{32} - 5) = \sqrt{2} \cdot \sqrt{32} - 5\sqrt{2}$ — **Distributive property**

$= \sqrt{64} - 5\sqrt{2}$ — **Product property of radicals**

$= 8 - 5\sqrt{2}$ — **Simplify.**

Use the **FOIL** method to multiply two binomials: **F**irst, **O**uter, **I**nner, **L**ast.

b. $(\sqrt{5} - \sqrt{3})(2\sqrt{5} + \sqrt{3})$

$= \sqrt{5} \cdot 2\sqrt{5} + \sqrt{5} \cdot \sqrt{3} + (-\sqrt{3})(2\sqrt{5}) + (-\sqrt{3})(\sqrt{3})$ — **Multiply.**

$= 2(\sqrt{5})^2 + \sqrt{5 \cdot 3} - 2\sqrt{3 \cdot 5} - (\sqrt{3})^2$ — **Product property of radicals**

$= 10 + \sqrt{15} - 2\sqrt{15} - 3$ — **Simplify.**

$= 7 - \sqrt{15}$ — **Simplify.**

PRACTICE **Simplify the radical expression.**

17. $5\sqrt{6} - \sqrt{3} + 2\sqrt{6}$

18. $9\sqrt{2} + 2\sqrt{32}$

19. $\sqrt{3}(5 + \sqrt{12})$

20. $8\sqrt{3} - \sqrt{75}$

21. $(\sqrt{6} - \sqrt{10})(\sqrt{6} + 2\sqrt{10})$

22. $(\sqrt{3} + 2\sqrt{2})(2\sqrt{3} - \sqrt{2})$

4. Solve a Radical Equation

Vocabulary

Radical equation An equation that contains a radical expression with a variable in the radicand.

EXAMPLE **Solve the radical equation.**

Squaring both sides of an equation: if $a = b$ then $a^2 = b^2$.

a. $5\sqrt{x} - 10 = 0$ — **Write original equation.**

$5\sqrt{x} = 10$ — **Add 10 to each side.**

$\sqrt{x} = 2$ — **Divide each side by 5.**

$(\sqrt{x})^2 = 2^2$ — **Square each side.**

$x = 4$ — **Simplify.**

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BENCHMARK 6

(Chapters 11, 12, and 13)

The radicand must be isolated on one side of the equation before you square each side.

Always check the solution by substituting it in the original equation.

When a radical equation contains two radical expressions, be sure that each side of the equation has only one radical expression before squaring each side.

b.

$5\sqrt{x-3}+6=21$	**Write original equation.**
$5\sqrt{x-3}=15$	**Subtract 6 from each side.**
$\sqrt{x-3}=3$	**Divide each side by 5.**
$(\sqrt{x-3})^2=3^2$	**Square each side.**
$x-3=9$	**Simplify.**
$x=12$	**Add 3 to each side.**

c.

$\sqrt{3x-7}=\sqrt{x+11}$	**Write original equation.**
$(\sqrt{3x-7})^2=(\sqrt{x+11})^2$	**Square each side.**
$3x-7=x+11$	**Simplify.**
$2x-7=11$	**Subtract x from each side.**
$2x=18$	**Add 7 to each side.**
$x=9$	**Simplify.**

PRACTICE **Solve the radical equation.**

23. $3\sqrt{x}-12=0$ **24.** $3\sqrt{x-8}+11=32$ **25.** $6\sqrt{x}-4=0$

26. $\sqrt{6x-15}=\sqrt{2x-3}$ **27.** $2\sqrt{x+9}-16=-6$ **28.** $\sqrt{5x-3}=\sqrt{2x+15}$

BENCHMARK 6

5. $4\sqrt{3}$ **6.** $\frac{\sqrt{11}}{5}$ **7.** $\frac{\sqrt{30x}}{5}$ **8.** $\frac{\sqrt{15}}{x^2}$ **9.** $\frac{2\sqrt{3x}}{z}$

10. $\frac{\sqrt{30a}}{3a}$ **11.** $3\sqrt{7}$ **12.** $4y\sqrt{y}$ **13.** $2z\sqrt{7y}$

14. $\frac{4\sqrt{3x}}{3x}$ **15.** $\frac{2\sqrt{5}}{3x}$ **16.** $\frac{5\sqrt{b}}{2b^2}$ **17.** $7\sqrt{6} - \sqrt{3}$

18. $17\sqrt{2}$ **19.** $5\sqrt{3} + 6$ **20.** $3\sqrt{3}$

21. $2\sqrt{15} - 14$ **22.** $2 + 3\sqrt{6}$ **23.** 16 **24.** 57

25. $\frac{4}{9}$ **26.** 3 **27.** 16 **28.** 6